Supplement to "Assessing Importance of Biomarkers: a Bayesian Joint Modeling Approach of Longitudinal and Survival Data with Semicompeting Risks"

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S.1 The Full Conditional Posterior Distributions

For (i), the full conditional distribution of $\boldsymbol{\gamma}$ given σ^2, Γ, $\boldsymbol{\theta}^{*R}$, $\boldsymbol{\theta}$, and D_{obs} is given by

$$\boldsymbol{\gamma}'|\sigma^2, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, D_{\text{obs}} \ \sim \ N(\boldsymbol{\mu}_{11}, V_{11}),$$

where $\boldsymbol{\mu}_{11} = V_{11}\big[\frac{\sum_{i=1}^n W_{i2}'(\boldsymbol{y}_i - W_{i1}(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta}))}{\sigma^2} + V_{01}^{-1}\boldsymbol{\mu}_{01}\big]$, and $V_{11} = [\sum_{i=1}^n W_{i2}'W_{i2} + V_{01}^{-1}]^{-1}$.

For (ii), $[\sigma^2|\boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, \Gamma, D_{\text{obs}}, \boldsymbol{\gamma}]$ is given by

$$\sigma^2|\boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, \Gamma, D_{\text{obs}}, \boldsymbol{\gamma} \sim \text{IG}(n + a_0, \frac{1}{2}\sum_{i=1}^n (\boldsymbol{y}_i - W_{i1}^*\boldsymbol{\theta}_i^* - W_i(\boldsymbol{\theta}', \boldsymbol{\gamma}')')'(\boldsymbol{y}_i - W_{i1}^*\boldsymbol{\theta}_i^* - W_i(\boldsymbol{\theta}', \boldsymbol{\gamma}')') + b_0).$$

For (iii), the density corresponding to $[a^*, b^*, c|\boldsymbol{\gamma}, \sigma^2, \boldsymbol{\theta}, \boldsymbol{\varphi}_{21}, \boldsymbol{\varphi}_3, \boldsymbol{\theta}^{*R}, D_{\text{obs}}]$ is given by

$$\begin{aligned}
&\pi((a^*, b^*, c)|\boldsymbol{\gamma}, \sigma^2, \boldsymbol{\theta}, \boldsymbol{\theta}^{*R}, D_{\text{obs}}) \\
\propto \ &\exp\Big\{ -\frac{1}{2\sigma^2}\sum_{i=1}^n\sum_{j=1}^{m_i}(y_{ij} - (\exp(a^*) + c * t_{ij})\theta_{i1}^* - (\exp(b^*) * t_{ij})\theta_{i2}^* - W_{ij}(\boldsymbol{\theta}', \boldsymbol{\gamma}')')^2) \\
&+ \sum_{i=1}^n \Big[1\{\zeta_i = 2,3\}\beta_1((\Gamma\boldsymbol{\theta}_i^*)'\boldsymbol{g}(t_{Pi})) - 1\{\zeta_i = 2,3\}\int_0^{t_{Pi}} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}du \\
&- 1\{\zeta_i = 1,4\}1\{\delta_i^* = 1\})\int_0^{t_i} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\} + 1\{\zeta_i = 1,2\}\beta_2(\Gamma\boldsymbol{\theta}_i^*)'\boldsymbol{g}(t) \\
&- \int_0^{t_i} \lambda_{20}(u)\exp\{\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u) + \boldsymbol{\alpha}_2'\boldsymbol{x}_i + \boldsymbol{\eta}'\boldsymbol{z}_i(u)\}du\Big] - \frac{1}{2}((a^*, b^*, c)' - \boldsymbol{\mu}_{06})'V_{06}^{-1}((a^*, b^*, c)' - \boldsymbol{\mu}_{06})\Big]\Big\}.
\end{aligned}$$

For (iv), when $i \in G_1, G_4$, $[\delta_i^*|\boldsymbol{\varphi}_2, \Gamma, \boldsymbol{\theta}_i^*, \boldsymbol{\theta}, D_{\text{obs}}]$ is a binomial distribution with

$$P(\delta_i^* = 1) = \frac{p(\boldsymbol{\phi}, \boldsymbol{x}_i) * P(T_P > t_i|\boldsymbol{\varphi}_{21}, \boldsymbol{x}_i, \Gamma, \boldsymbol{\theta}_i^*, \boldsymbol{\theta}, \delta = 1)}{1 - p(\boldsymbol{\phi}, \boldsymbol{x}_i) + p(\boldsymbol{\phi}, \boldsymbol{x}_i) * P(T_P > t_i|\boldsymbol{\varphi}_{21}, \boldsymbol{x}_i, \Gamma, \boldsymbol{\theta}_i^*, \boldsymbol{\theta}, \delta = 1)}.$$

For (v), the density for $[\boldsymbol{\phi}|\boldsymbol{\delta}^*, D_{\text{obs}}]$ is given by

$$\pi(\boldsymbol{\phi}|\boldsymbol{\delta}^*, D_{\text{obs}}) \propto \Big[-\frac{1}{2}(\boldsymbol{\phi}-\boldsymbol{\mu}_{02})'V_{02}^{-1}(\boldsymbol{\phi}'-\boldsymbol{\mu}_{02})\Big] \\ \times [p(\boldsymbol{\phi},\boldsymbol{x}_i)]^{\sum_{i=1}^n(1\{\zeta_i=2,3\}+\delta_i^*1\{\zeta_i=1,4\})}[1-p(\boldsymbol{\phi},\boldsymbol{x}_i)]^{\sum_{i=1}^n(1-\delta_i^*)1\{\zeta_i=1,4\}}.$$

For (vi), the density for $[\boldsymbol{\beta}_1, \boldsymbol{\alpha}_1|\boldsymbol{\lambda}_1, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, \boldsymbol{\delta}^*, D_{\text{obs}}]$ is given by

$$\pi(\boldsymbol{\beta}_1, \boldsymbol{\alpha}_1|\ \boldsymbol{\lambda}_1, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, \boldsymbol{\delta}^*, D_{\text{obs}}) \\ \propto \exp\Big[-\frac{1}{2}((\boldsymbol{\beta}_1', \boldsymbol{\alpha}_1')'-\boldsymbol{\mu}_{04})'V_{04}^{-1}((\boldsymbol{\beta}_1', \boldsymbol{\alpha}_1')'-\boldsymbol{\mu}_{04}) + \sum_{i=1}^n 1\{\zeta_i=2,3\}(\beta_1((\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(t_{Pi})) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i) \\ - \sum_{i=1}^n 1\{\zeta_i=2,3\}\int_0^{t_{Pi}} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}du \\ - \sum_{i=1}^n 1\{\zeta_i=1,4\}1\{\delta_i^*=1\})\int_0^{t_i} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}].$$

For (vii), given $(\boldsymbol{\beta}_1, \boldsymbol{\alpha}_1, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, \boldsymbol{\delta}^*, D_{\text{obs}})$, the λ_{1k}'s are conditionally independent. The density for $[\boldsymbol{\lambda}_1|\boldsymbol{\beta}_1, \boldsymbol{\alpha}_1, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, \boldsymbol{\delta}^*, D_{\text{obs}}]$ is given by

$$\pi(\boldsymbol{\lambda}_1|\boldsymbol{\beta}_1, \boldsymbol{\alpha}_1, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, \boldsymbol{\delta}^*, D_{\text{obs}}) \\ \propto \prod_{k=1}^{K_1}\lambda_{1k}^{a_1-1}\exp(-b_1\lambda_{1k})\prod_{i=1}^n \lambda_{10}(t_{Pi})^{1\{\zeta_i=2,3\}}\exp\Big[\sum_{i=1}^n 1\{\zeta_i=2,3\}\int_0^{t_{Pi}}\lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})' \\ \boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}du - \sum_{i=1}^n 1\{\zeta_i=1,4\}1\{\delta_i^*=1\})\int_0^{t_i}\lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}\Big].$$

With the use of Riemann approximation to the integral, we have

$$\lambda_{1k}|\boldsymbol{\alpha}_1, \boldsymbol{\beta}_1, \boldsymbol{\theta}^{*R}, D_{\text{obs}}, \boldsymbol{\delta}^* \sim \text{Gamma}(a_{1k}, b_{1k}),$$

where $a_{1k} = a_1 + \sum_{i=1}^n 1\{\zeta_i=2,3\}1\{s_{1,k-1} < t_{S_i} \le s_{1,k}\}$, $b_{1k} = b_1 + \sum_{i=1}^n \exp\{\boldsymbol{\beta}_1'\boldsymbol{\theta}_i^* + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}\big[H_{1k}(t_{S_i})1\{\zeta_i=2,3\} + H_{1k}(t_i)1\{\zeta_i=1,4\}1\{\delta_i^*=1\}\big]$, $H_{1k}(t) = 1\{t > s_{1,k}\}\sum_{l=1}^{N_1}\frac{s_{1,k}-s_{1,k-1}}{N_1}\exp\{\beta_1(\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(w(s_{1,k-1}, s_{1,k}, l))\} + 1\{s_{1,k-1} < t \le s_{1,k}\}\sum_{l=1}^{N_1}\frac{t-s_{1,k-1}}{N_1}\exp\{\beta_1(\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(w(s_{1,k-1}, t, l))\}$, $w(u_1, u_2, l) = u_1 + \frac{(u_2-u_1)l}{N_1}$.

For (viii), the density for $[\boldsymbol{\beta}_2, \boldsymbol{\alpha}_2, \boldsymbol{\eta}|\boldsymbol{\lambda}_2, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, D_{\text{obs}}]$ is given by

$$\pi(\boldsymbol{\beta}_2, \boldsymbol{\alpha}_2, \boldsymbol{\eta}|\boldsymbol{\lambda}_2, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, D_{\text{obs}}) \propto \exp\Big\{\sum_{i=1}^n\Big[1\{\zeta_i=1,2\}(\beta_2(\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(t) + \boldsymbol{\alpha}_2'\boldsymbol{x}_i + \boldsymbol{\eta}'\boldsymbol{z}_i(t_i)) \\ - \int_0^{t_i}\lambda_{20}(u)\exp\{\beta_2(\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(u) + \boldsymbol{\alpha}_2'\boldsymbol{x}_i + \boldsymbol{\eta}'\boldsymbol{z}_i(u)\}du\Big]\Big\}.$$

For (ix), given $(\boldsymbol{\alpha}_2, \boldsymbol{\beta}_2, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, D_{\text{obs}})$, the λ_{2k}'s are conditionally independent. The density for $[\boldsymbol{\lambda}_2|\boldsymbol{\alpha}_2, \boldsymbol{\beta}_2, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, D_{\text{obs}}]$ is given by

$$\pi(\boldsymbol{\lambda}_2|\boldsymbol{\alpha}_2, \boldsymbol{\beta}_2, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, D_{\text{obs}}) \propto \prod_{k=1}^{K_2}\lambda_{2k}^{a_2-1}\exp(-b_2\lambda_{2k})\prod_{i=1}^n\lambda_{20}(t_i)^{1\{\zeta_i=1,2\}}\exp\Big\{-\sum_{i=1}^n\Big[\int_0^{t_i}\lambda_{20}(u)\exp\{ \\ \beta_2(\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(u) + \boldsymbol{\alpha}_2'\boldsymbol{x}_i + \boldsymbol{\eta}'\boldsymbol{z}_i(u)\}du\Big]\Big\}.$$

After the use of Riemann approximation to the integral, we have

$$\lambda_{2k}|\boldsymbol{\alpha}_2, \boldsymbol{\beta}_2, \Gamma, \boldsymbol{\theta}^{*R}, \boldsymbol{\theta}, D_{\text{obs}} \sim \text{Gamma}(a_{2k}, b_{2k}),$$

where $a_{2k} = a_2 + \sum_{i=1}^n 1\{\zeta_i = 1,2\}1\{s_{2,k-1} < t_i \le s_{2,k}\}$, $b_{2k} = b_2 + \sum_{i=1}^n \exp\{\boldsymbol{\beta}_2'\boldsymbol{\theta}_i^* + \boldsymbol{\alpha}_2'\boldsymbol{x}_i\}H_{2k}(t_i)$, $H_{2k}(t) = 1\{t > s_{2,k}\}\sum_{l=1}^{N_1} \frac{s_{2,k}-s_{2,k-1}}{N_1} \exp\{\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(w(s_{2,k-1}, s_{2,k}, l)) + \boldsymbol{z}_i(w(s_{2,k-1}, s_{2,k}, l))\} + 1\{s_{2,k-1} < t \le s_{2,k}\}\sum_{l=1}^{N_1} \frac{t-s_{2,k-1}}{N_1} \exp\{\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(w(s_{2,k-1}, t, l)) + \boldsymbol{z}_i(w(s_{2,k-1}, t, l))\}$, $w(u_1, u_2, l) = u_1 + \frac{(u_2-u_1)l}{N_1}$, for $k = 1, \dots, K_2$.

For (x), the density for $[\boldsymbol{\theta}_i^*|\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_{21}, \boldsymbol{\varphi}_3, \boldsymbol{\delta}^*, D_{\text{obs}}]$ is given by

$$\begin{aligned}
&\pi(\boldsymbol{\theta}_i^*|\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_{21}, \boldsymbol{\varphi}_3, \boldsymbol{\delta}^*, D_{\text{obs}}) \\
&\propto \exp\Big[-\frac{1}{2\sigma^2}(\boldsymbol{y}_i - W_{i1}^*\boldsymbol{\theta}_i^* - W_i(\boldsymbol{\theta}', \boldsymbol{\gamma}')')'(\boldsymbol{y}_i - W_{i1}^*\boldsymbol{\theta}_i^* - W_i(\boldsymbol{\theta}', \boldsymbol{\gamma}')') - \frac{1}{2}(\boldsymbol{\theta}_i^*)'\boldsymbol{\theta}_i^* \\
&\quad + 1\{\zeta_i = 2,3\}\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(t_{Pi})) - 1\{\zeta_i = 2,3\}\int_0^{t_{Pi}} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}du \\
&\quad - 1\{\zeta_i = 1,4\}1\{\delta_i^* = 1\})\int_0^{t_i} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\} + 1\{\zeta_i = 1,2\}\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(t) \\
&\quad - \int_0^{t_i} \lambda_{20}(u)\exp\{\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u) + \boldsymbol{\alpha}_2'\boldsymbol{x}_i + \boldsymbol{\eta}'\boldsymbol{z}_i(u)\}du\Big].
\end{aligned}$$

For (xi), the density for $[\boldsymbol{\theta}|\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_{21}, \boldsymbol{\varphi}_3, \boldsymbol{\delta}^*, \boldsymbol{\theta}^{*R}, D_{\text{obs}}]$ is given by

$$\begin{aligned}
&\pi(\boldsymbol{\theta}|\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_{21}, \boldsymbol{\varphi}_3, \boldsymbol{\delta}^*, \boldsymbol{\theta}^{*R}, D_{\text{obs}}) \\
&\propto \exp\Big\{ -\frac{1}{2}(\boldsymbol{\theta} - \boldsymbol{\mu}_{02})'V_{02}^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu}_{02}) + \sum_{i=1}^n \Big[-\frac{1}{2\sigma^2}(\boldsymbol{y}_i - W_{i1}^*\boldsymbol{\theta}_i^* - W_i(\boldsymbol{\theta}', \boldsymbol{\gamma}')')'(\boldsymbol{y}_i - W_{i1}^*\boldsymbol{\theta}_i^* - W_i(\boldsymbol{\theta}', \boldsymbol{\gamma}')') \\
&\quad + 1\{\zeta_i = 2,3\}\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(t_{Pi})) - 1\{\zeta_i = 2,3\}\int_0^{t_{Pi}} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\}du \\
&\quad - 1\{\zeta_i = 1,4\}1\{\delta_i^* = 1\})\int_0^{t_i} \lambda_{10}(u)\exp\{\beta_1((\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u)) + \boldsymbol{\alpha}_1'\boldsymbol{x}_i\} + 1\{\zeta_i = 1,2\}\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(t) \\
&\quad - \int_0^{t_i} \lambda_{20}(u)\exp\{\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(u) + \boldsymbol{\alpha}_2'\boldsymbol{x}_i + \boldsymbol{\eta}'\boldsymbol{z}_i(u)\}du\Big]\Big\}.
\end{aligned}$$

S.2 Results and Formulas

S.2.1 Result 1: CPO decomposition identities

For CPO_i, $\text{CPO}_{i,\text{Long}}$, $\text{CPO}_{i,\text{Pg}|\text{Long}}$, and $\text{CPO}_{i,\text{Surv}|\text{Pg},\text{Long}}$, we have the following identities:

$$
\begin{aligned}
\frac{\pi(\boldsymbol{\varphi}_1^*|D_{\text{obs}}^{(-i)})}{\pi(\boldsymbol{\varphi}_1^*|D_{\text{obs}})} &= \frac{\int \pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3|D_{\text{obs}}^{(-i)}) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3}{\int \pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3|D_{\text{obs}}) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3} \\
&= \frac{\int \frac{\prod_{j\neq i} f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3}{c(D_{\text{obs}}^{(-i)})}}{\int \frac{\prod_{j=1}^n f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3}{c(D_{\text{obs}})}} \\
&= \text{CPO}_i \int \frac{1}{f(\boldsymbol{y}_i, t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i)} \\
&\quad \times \frac{\prod_{j=1}^n f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3)}{\int \prod_{j=1}^n f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3} d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3 \\
&= \text{CPO}_i \int \frac{1}{f(\boldsymbol{y}_i, t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i)} \pi(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*, D_{\text{obs}}) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3,
\end{aligned}
$$

$$
\begin{aligned}
\text{CPO}_{i,\text{Long}} &= \text{CPO}_i f(\boldsymbol{y}_i|\boldsymbol{\varphi}_1^*, \boldsymbol{x}_i) \int \frac{1}{f(\boldsymbol{y}_i, t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i)} \pi(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*, D_{\text{obs}}) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3 \\
&= \text{CPO}_i \int \frac{f(\boldsymbol{y}_i|\boldsymbol{\varphi}_1^*, \boldsymbol{x}_i)}{f(\boldsymbol{y}_i|\boldsymbol{\varphi}_1^*, \boldsymbol{x}_i) f(t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i, \boldsymbol{y}_i)} \pi(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*, D_{\text{obs}}) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3 \\
&= \text{CPO}_i \int \frac{1}{f(t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i, \boldsymbol{y}_i)} \pi(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*, D_{\text{obs}}) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3,
\end{aligned}
$$

$$
\begin{aligned}
\frac{\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*|D_{\text{obs}}^{(-i)})}{\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*|D_{\text{obs}})} &= \frac{\int \pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3|D_{\text{obs}}^{(-i)}) d\boldsymbol{\varphi}_3}{\int \pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3|D_{\text{obs}}) d\boldsymbol{\varphi}_3} \\
&= \frac{\int \frac{\prod_{j\neq i} f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3) d\boldsymbol{\varphi}_3}{c(D_{\text{obs}}^{(-i)})}}{\int \frac{\prod_{j=1}^n f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3) d\boldsymbol{\varphi}_3}{c(D_{\text{obs}})}} \\
&= \text{CPO}_i \int \frac{1}{f(\boldsymbol{y}_i, t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i)} \\
&\quad \times \frac{\prod_{j=1}^n f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3)}{\int \prod_{j=1}^n f(\boldsymbol{y}_j, t_{Pj}, \delta_j, t_j|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3, \boldsymbol{x}_j, \nu_j)\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3) d\boldsymbol{\varphi}_3} d\boldsymbol{\varphi}_3 \\
&= \text{CPO}_i \int \frac{1}{f(\boldsymbol{y}_i, t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i)} \pi(\boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, D_{\text{obs}}) d\boldsymbol{\varphi}_3,
\end{aligned}
$$

$$
\begin{aligned}
\text{CPO}_{i,\text{Pg}|\text{Long}} &= \frac{f(t_{Pi}, \delta_i|\boldsymbol{y}_i, \boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{x}_i) \frac{\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*|D_{\text{obs}}^{(-i)})}{\pi(\boldsymbol{\varphi}_1^*|D_{\text{obs}}^{(-i)})}}{\frac{\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*|D_{\text{obs}})}{\pi(\boldsymbol{\varphi}_1^*|D_{\text{obs}})}} \\
&= f(t_{Pi}, \delta_i|\boldsymbol{y}_i, \boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{x}_i) [\frac{\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*|D_{\text{obs}}^{(-i)})}{\pi(\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*|D_{\text{obs}})}] [\frac{\pi(\boldsymbol{\varphi}_1^*|D_{\text{obs}})}{\pi(\boldsymbol{\varphi}_1^*|D_{\text{obs}}^{(-i)})}] \\
&= f(t_{Pi}, \delta_i|\boldsymbol{y}_i, \boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{x}_i) \frac{[\int \frac{1}{f(\boldsymbol{y}_i, t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i)} \pi(\boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2^*, D_{\text{obs}}) d\boldsymbol{\varphi}_3]}{[\int \frac{1}{f(\boldsymbol{y}_i, t_{Pi}, \delta_i, t_i|\boldsymbol{\varphi}_1^*, \boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{z}_i(t), \nu_i)} \pi(\boldsymbol{\varphi}_2, \boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*, D_{\text{obs}}) d\boldsymbol{\varphi}_2 d\boldsymbol{\varphi}_3]}.
\end{aligned}
$$

From $\text{CPO}_i = \text{CPO}_{i,\text{Long}} * \text{CPO}_{i,\text{Pg|Long}} * \text{CPO}_{i,\text{Surv|Pg,Long}}$, we have

$$\begin{aligned}\text{CPO}_{i,\text{Surv|Pg,Long}} &= \frac{1}{f(t_{Pi},\delta_i,\boldsymbol{y}_i|\boldsymbol{\varphi}_1^*,\boldsymbol{\varphi}_2^*,\boldsymbol{x}_i)[\int \frac{1}{f(t_i,t_{Pi},\delta_i,\boldsymbol{y}_i|\boldsymbol{\varphi}_1^*,\boldsymbol{\varphi}_2^*,\boldsymbol{\varphi}_3,\boldsymbol{x}_i,\nu_i)}\pi(\boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*,\boldsymbol{\varphi}_2^*,D_{\text{obs}})d\boldsymbol{\varphi}_3]} \\ &= \frac{1}{[\int \frac{1}{f(t_i|\boldsymbol{\varphi}_1^*,\boldsymbol{\varphi}_2^*,\boldsymbol{\varphi}_3,\boldsymbol{x}_i,t_{Pi},\delta_i,\nu_i,\boldsymbol{y}_i)}\pi(\boldsymbol{\varphi}_3|\boldsymbol{\varphi}_1^*,\boldsymbol{\varphi}_2^*,D_{\text{obs}})d\boldsymbol{\varphi}_3]}.\end{aligned}$$

S.2.2 Result 2: Weight Function

According to Zhang et al. (2017), Let $w_i(\boldsymbol{\theta}_i)$ be a normalized weight function such that $\int w_i(\boldsymbol{\theta}_i)d\boldsymbol{\theta}_i = 1$. Then, we have

$$\begin{aligned}\text{CPO}_i &= \frac{c(D_{\text{obs}})}{\int \prod_{j\neq i} f(\boldsymbol{y}_j,t_{Sj},\delta_j,t_j,\boldsymbol{\theta}_j^*|\boldsymbol{\varphi},\boldsymbol{x}_j,\nu_j)\pi(\boldsymbol{\varphi})d\boldsymbol{\theta}^{*R}_{(-i)}d\boldsymbol{\varphi}} \\ &= \frac{c(D_{\text{obs}})}{\int w_i(\boldsymbol{\theta}_i)\prod_{j\neq i} f(\boldsymbol{y}_j,t_{Sj},\delta_j,,t_j,\boldsymbol{\theta}_j^*|\boldsymbol{\varphi},\boldsymbol{x}_j,\nu_j)\pi(\boldsymbol{\varphi})d\boldsymbol{\theta}_i d\boldsymbol{\theta}^{*R}_{(-i)}d\boldsymbol{\varphi}} \\ &= \frac{1}{\int \frac{w_i(\boldsymbol{\theta}_i)}{f(\boldsymbol{y}_i,t_{Si},\delta_i,t_i,\boldsymbol{\theta}_i|\boldsymbol{\varphi},\boldsymbol{x}_i,\boldsymbol{z}_i(t),\nu_i)}\frac{\prod_{j=1}^n f(\boldsymbol{y}_j,t_{Sj},\delta_j,t_j,\boldsymbol{\theta}_j^*|\boldsymbol{\varphi},\boldsymbol{x}_j,\nu_j)\pi(\boldsymbol{\varphi})}{c(D_{\text{obs}})}d\boldsymbol{\theta}_i d\boldsymbol{\theta}^{*R}_{(-i)}d\boldsymbol{\varphi}} \\ &= \frac{1}{\int \frac{w_i(\boldsymbol{\theta}_i^*)}{f(\boldsymbol{y}_i,t_{Si},\delta_i,t_i,\boldsymbol{\theta}_i^*|\boldsymbol{\varphi},\boldsymbol{x}_i,\boldsymbol{z}_i(t),\nu_i)}\pi(\boldsymbol{\varphi},\boldsymbol{\theta}^{*R}|D_{\text{obs}})d\boldsymbol{\theta}^{*R}d\boldsymbol{\varphi}}.\end{aligned}$$

The optimal choice of w_i in minimizing the variance of $\frac{w_i(\boldsymbol{\theta}_{ib}^*)}{f(\boldsymbol{y}_i,t_{Pi},\delta_i,t_i,\boldsymbol{\theta}_{ib}^*|\boldsymbol{\varphi}_b,\boldsymbol{x}_i,\boldsymbol{z}_i(t),\nu_i)}$ in $\widehat{\text{CPO}}_i$ is $w_{i,\text{opt}}(\boldsymbol{\theta}_i^*) = \frac{f(\boldsymbol{y}_i,t_{Si},\delta_i,t_i,\boldsymbol{\theta}_i^*|\boldsymbol{\varphi},\boldsymbol{x}_i,\boldsymbol{z}_i(t),\nu_i)}{f(\boldsymbol{y}_i,t_{Si},\delta_i,t_i|\boldsymbol{\varphi},\boldsymbol{x}_i,\boldsymbol{z}_i(t),\nu_i)}$. A reasonable choice of $w_i(\boldsymbol{\theta}_i)$ is a multivariate normal density, which can be constructed by the Laplace approximation of the joint density $f(\boldsymbol{y}_i,t_{Si},\delta_i,t_i,\boldsymbol{\theta}_i^*|\boldsymbol{\varphi},\boldsymbol{x}_i,\boldsymbol{z}_i(t),\nu_i)$. Another possible choice is $w_{i,\text{cond}}(\boldsymbol{\theta}_i^*) = f(\boldsymbol{\theta}_i^*|\boldsymbol{\varphi}_1,\boldsymbol{x}_i,\boldsymbol{y}_i)$. The conditional distribution of $\boldsymbol{\theta}_i^*|\boldsymbol{\varphi}_1,\boldsymbol{x}_i,\boldsymbol{y}_i$ is $\boldsymbol{\theta}_i^*|\boldsymbol{\varphi}_1,\boldsymbol{x}_i,\boldsymbol{y}_i \sim N(\boldsymbol{\mu}^*,\Sigma^*)$, where $\boldsymbol{\mu}^* = \Gamma' W_{i1}'(\sigma^2 I_{m_i} + W_{i1}\Gamma\Gamma' W_{i1}')^{-1}(\boldsymbol{y}_i - W_i(\boldsymbol{\theta}',\boldsymbol{\gamma}')')$ and $\Sigma^* = I_p - \Gamma' W_{i1}'(\sigma^2 I_{m_i} + W_{i1}\Gamma\Gamma' W_{i1}')^{-1}W_{i1}\Gamma$.

References

Zhang, D., Chen, M.-H., Ibrahim, J. G., Boye, M. E., and Shen, W. (2017). Bayesian model assessment in joint modeling of longitudinal and survival data with applications to cancer clinical trials. *Journal of Computational and Graphical Statistics*, 26(1):121–133.

S.2.3 Result 3: Numeric Approximation

For Riemann approximation to the integral, we assume $N_1 = 30$. Denote function $w(t_1,t_2,l) = t_1 + \frac{t_2-t_1}{N_1}l$, where $l = 1,\dots,N_1$. Approximate $S_{1i}(t|\boldsymbol{\lambda}_1,\beta_1,\Gamma,\boldsymbol{\theta}_i^*,\boldsymbol{\theta}) = \sum_{k=1}^{K_1}\Big[1\{s_{1,k-1} < t \le s_{1,k}\} \times \{\lambda_{1k}\sum_{l=1}^{N_1}\frac{t-s_{1,k-1}}{N_1}\exp\{\beta_1(\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}(w(s_{1,k-1},t,l))\} + \sum_{g=1}^{k-1}\lambda_{1g}\sum_{l=1}^{N_1}\frac{s_{1,g}-s_{1,g-1}}{N_1}\exp\{\beta_1(\Gamma\boldsymbol{\theta}_i^*+\boldsymbol{\theta})'\boldsymbol{g}($

$w(s_{1,g-1}, s_{1,g}, l))\}\Big]$ and $S_{i2}(t|\boldsymbol{\lambda}_2, \beta_2, \Gamma, \boldsymbol{\theta}_i^*, \boldsymbol{\theta}) = \sum_{k=1}^{K_1}\Big[1\{s_{2,k-1} < t \le s_{2,k}\} \times \{\lambda_{2k}\sum_{l=1}^{N_1}\frac{t-s_{2,k-1}}{N_1}\exp\{\beta_2$ $(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(w(s_{2,k-1}, t, l))\} + \sum_{g=1}^{k-1}\lambda_{2g}\sum_{l=1}^{N_1}\frac{s_{2,g}-s_{2,g-1}}{N_1}\exp\{\beta_2(\Gamma\boldsymbol{\theta}_i^* + \boldsymbol{\theta})'\boldsymbol{g}(w(s_{2,g-1}, s_{2,g}, l))\}\Big]$.

S.3 A Simulation Study

We perform a simulation study to evaluate the estimation of the parameters in the joint modeling. Baseline covariates X_1 and X_2 are independently generated from $N(0,1)$ and Bernoulli($p = .5$). Treatment $X_3 = 1$ for half of the subjects. $\boldsymbol{x} = (X_1, X_2, X_3)$ In longitudinal model, $\boldsymbol{\theta}_i^* = (\boldsymbol{\theta}_{i0}^*, \boldsymbol{\theta}_{i1}^*) \sim N(0, \Omega)$ independently, $\Omega = \begin{pmatrix} 2.0 & -0.3 \\ -0.3 & 4.5 \end{pmatrix}$. Let $\boldsymbol{\theta} = (4.8, 1.7)$, and $\boldsymbol{\theta}_i = \boldsymbol{\theta}_i^* + \boldsymbol{\theta}$. We then simulate the longitudinal data from a $N(\mu_i(a_{ij}), \sigma^2)$ with a linera trajectory $\mu_i(a_{ij}) = \boldsymbol{\theta}_{i0} + a_{ij}\boldsymbol{\theta}_{i1} + \boldsymbol{\gamma}'\boldsymbol{x}_i$. Measurement time a_{ij} is generated with gap around six weeks up to five measures. $\sigma^2 = 0.6$ and $\boldsymbol{\gamma} = (1.2, -0.1, -0.1)$.

We generate the indicator δ for whether the subject will have progression eventually from a logistic model $\text{logit}(P(\delta = 1) = 2.0 - 1.38X_1 + 1.233X_2 + 0.32X_3$. Simulate progression time T_P value if $\delta = 1$. Baseline hazard rate $\lambda_{10} = 2.5$, coefficients $\beta_1 = -0.02$ and $\boldsymbol{\alpha}_1 = (0.29, 0.45, 0.01)$. Denote $\mu_i^*(t) = \boldsymbol{\theta}_{i0} + \boldsymbol{\theta}_{i1}(t * 1(t \le \text{Tmax}) + \text{Tmax} * 1(t > \text{Tmax}))$, where we assume the trajectory $\mu_i^*(t)$ won't change after Tmax $= .48$. We can write the survival function for T_P as $P(T_P > t|\boldsymbol{\varphi}_{21}, \boldsymbol{x}_i, \boldsymbol{\theta}_i, \delta = 1) = \exp[-\lambda_{10}\exp\{\boldsymbol{\alpha}_1'\boldsymbol{x}_i\} * \int_0^t \exp\{\beta_1\mu_i^*(u)\}du$. Then we use the inverse cumulative distribution function (cdf) method to generate T_P.

For time to death T_D, we assume baseline hazard rate $\lambda_{20} = 0.45$, coefficients $\boldsymbol{\alpha}_2 = (0.8, -0.3, -0.25)$, $\beta_2 = 0.3$, and $\boldsymbol{\eta} = (0.01, 0.2)$. If $\delta = 0$, $P(T_D > t|\boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{\theta}_i, t_{Pi}, \delta = 1) = \exp[-\lambda_{20}\exp\{\boldsymbol{\alpha}_2'\boldsymbol{x}_i\} * \int_0^t \exp\{\beta_2\mu_i^*(u)\}du)]$. If $\delta = 1$,

$$P(T_D > t|\boldsymbol{\varphi}_3, \boldsymbol{x}_i, \boldsymbol{\theta}_i, t_{Pi}, \delta = 1) = \begin{cases} \exp[-\lambda_{20}\exp\{\boldsymbol{\alpha}_2'\boldsymbol{x}_i\} * \int_0^t \exp\{\beta_2\mu_i^*(u)\}du)] & \text{if } t \le t_{Pi} \\ \exp[-\lambda_{20}\exp\{\boldsymbol{\alpha}_2'\boldsymbol{x}_i\} * (\int_0^{t_{Pi}} \exp\{\beta_2\mu_i^*(u)\}du + \\ \qquad \exp(\eta_1 + \eta_2 X_3)\int_{t_{Pi}}^t \exp\{\beta_2\mu_i^*(u)\}du)] & \text{if } t > t_{Pi} \end{cases}$$

Then we use the inverse cdf method to generate T_D. T_D is censored by an exponential distribution with rate 0.2. Both T_P and the longitudinal measures are censored by T_D.

Table S.1: Simulation Results

Par	TRUE	EST	SD	SE	MSE	CP	Par	TRUE	EST	SD	SE	MSE	CP
$\boldsymbol{\alpha}_2$	0.800	0.808	0.057	0.056	0.056	97	a	1.414	1.421	0.059	0.057	0.057	96
	-0.300	-0.296	0.109	0.111	0.111	93	b	2.111	2.050	0.215	0.224	0.232	94
	-0.250	-0.243	0.159	0.167	0.167	92	c	-0.212	-0.200	0.220	0.226	0.226	92
$\boldsymbol{\eta}$	0.010	0.023	0.174	0.171	0.171	96	Ω_{11}	2.000	2.027	0.169	0.162	0.164	96
	0.200	0.198	0.209	0.206	0.206	96	Ω_{12}	-0.300	-0.293	0.318	0.325	0.324	92
β_2	0.300	0.297	0.037	0.034	0.034	96	Ω_{22}	4.500	4.438	0.915	0.955	0.954	94
λ_2	0.450	0.473	0.112	0.107	0.109	94	σ^2	0.600	0.604	0.030	0.031	0.031	93
$\boldsymbol{\alpha}_1$	0.290	0.249	0.088	0.084	0.093	92	$\boldsymbol{\theta}$	4.800	4.804	0.143	0.140	0.140	94
	0.450	0.457	0.158	0.160	0.159	94		1.700	1.699	0.194	0.180	0.179	97
	0.010	-0.023	0.157	0.160	0.163	94	$\boldsymbol{\gamma}$	1.200	1.207	0.073	0.070	0.070	97
β_1	-0.020	-0.017	0.048	0.049	0.049	94		-0.100	-0.100	0.142	0.150	0.149	93
λ_1	2.500	2.650	0.770	0.777	0.789	96		-0.100	-0.114	0.151	0.159	0.159	92
$\boldsymbol{\phi}$	2.000	2.125	0.804	0.596	0.608	100							
	-1.380	-1.130	0.510	0.412	0.481	94							
	1.233	1.133	1.032	0.692	0.697	99							
	0.320	1.591	1.652	1.152	1.714	98							

The average of the posterior means (EST), and the average of the posterior standard deviations (SD), the simulation standard error (SE), the root of the mean squared error (RMSE), and the coverage probability (CP) of the 95% highest posterior density (HPD) intervals for each parameter are calculated over the 200 simulated datasets. The results are reported in Table S.1. We see from Table S.1 that (i) the posterior estimates are close to the true values for almost all of the parameters except for $\boldsymbol{\phi}$; (ii) the SD's are close to the SE's for most of the parameters; and (iii) the CP's are close to 95% for most of the parameters. We note that the parameters $\boldsymbol{\phi}$ are less identifiable than the other parameters since we assume that "cure" is not observed.

S.4 Additional Tables

Table S.2: Parameter Estimates under the Joint Model with GH

Parameter	EST	SD	95%HPD	Parameter	EST	SD	95%HPD
Survival				Progression			
ECOG	0.787	0.145	(0.512 , 1.081)	ECOG	0.048	0.159	(-0.248 , 0.366)
AGE	-0.074	0.138	(-0.347 , 0.188)	AGE	-0.200	0.188	(-0.550 , 0.174)
PEGFRAD	0.152	0.127	(-0.088 , 0.406)	PEGFRAD	0.427	0.144	(0.157 , 0.714)
Female	-0.022	0.169	(-0.346 , 0.319)	Female	-0.058	0.207	(-0.444 , 0.369)
PCRTDC	0.209	0.120	(-0.022 , 0.449)	PCRTDC	0.016	0.149	(-0.288 , 0.299)
TMSITED01	0.401	0.135	(0.139 , 0.666)	TMSITED01	-0.170	0.148	(-0.472 , 0.116)
METLOC2	0.059	0.146	(-0.227 , 0.342)	METLOC2	-0.071	0.149	(-0.377 , 0.206)
METLOC3	-0.289	0.129	(-0.553 , -0.053)	METLOC3	0.035	0.140	(-0.223 , 0.329)
METLOC4	0.510	0.202	(0.129 , 0.916)	METLOC4	0.658	0.229	(0.203 , 1.101)
METLOC5	0.312	0.210	(-0.095 , 0.720)	METLOC5	0.092	0.237	(-0.362 , 0.564)
BSDTARG	0.009	0.001	(0.006 , 0.012)	BSDTARG	0.003	0.002	(0.000 , 0.006)
R	-0.363	0.165	(-0.691 , -0.057)	R	-0.039	0.150	(-0.324 , 0.253)
$\mathrm{V}_{\mathrm{T_P}}$	0.902	0.140	(0.644 , 1.189)	β_1	0.148	0.050	(0.052 , 0.246)
$\mathrm{V}_{\mathrm{T_{SW}}}$	-0.266	0.202	(-0.664 , 0.121)	Longitudinal			
$\mathrm{R} * \mathrm{V}_{\mathrm{T_{SW}}}$	0.316	0.234	(-0.149 , 0.750)	θ_1	4.863	0.271	(4.367 , 5.397)
β_2	0.296	0.052	(0.195 , 0.393)	θ_2	0.126	0.024	(0.080 , 0.173)
Logistic				ECOG	1.285	0.177	(0.940 , 1.626)
Intercept	2.949	1.190	(0.960 , 5.320)	AGE	0.187	0.176	(-0.144 , 0.539)
ECOG	-0.033	0.743	(-2.019 , 2.011)	PEGFRAD	0.263	0.153	(-0.021 , 0.578)
AGE	-0.271	0.831	(-2.265 , 2.809)	Female	0.113	0.219	(-0.326 , 0.534)
PEGFRAD	-0.574	0.834	(-2.884 , 1.549)	PCRTDC	0.102	0.154	(-0.189 , 0.415)
Female	0.668	1.329	(-2.169 , 4.564)	TMSITED01	0.204	0.169	(-0.131 , 0.520)
PCRTDC	1.093	1.314	(-0.806 , 3.200)	METLOC2	0.169	0.187	(-0.196 , 0.537)
R	1.439	1.702	(-0.783 , 2.239)	METLOC3	-0.028	0.158	(-0.342 , 0.267)
				METLOC4	-0.604	0.276	(-1.182 , -0.093)
				METLOC5	0.339	0.282	(-0.197 , 0.898)
				BSDTARG	0.002	0.002	(-0.002 , 0.006)
				R	-0.029	0.162	(-0.331 , 0.295)
				Ω_{11}	2.036	0.211	(1.644 , 2.463)
				Ω_{21}	-0.021	0.040	(-0.097 , 0.059)
				Ω_{22}	0.028	0.016	(0.000 , 0.057)

Table S.3: Posterior Estimates under the Joint Model with HNPA

Parameter	EST	SD	95%HPD	Parameter	EST	SD	95%HPD
Survival				Progression			
ECOG	0.705	0.139	(0.436 , 0.977)	ECOG	0.023	0.155	(-0.268 , 0.339)
AGE	-0.080	0.128	(-0.343 , 0.158)	AGE	-0.209	0.187	(-0.564 , 0.156)
PEGFRAD	0.062	0.118	(-0.164 , 0.291)	PEGFRAD	0.445	0.143	(0.189 , 0.750)
Female	-0.111	0.163	(-0.439 , 0.201)	Female	-0.075	0.209	(-0.479 , 0.340)
PCRTDC	0.188	0.115	(-0.045 , 0.408)	PCRTDC	0.031	0.142	(-0.248 , 0.307)
TMSITED01	0.395	0.127	(0.150 , 0.646)	TMSITED01	-0.154	0.147	(-0.428 , 0.145)
METLOC2	0.104	0.137	(-0.179 , 0.363)	METLOC2	-0.072	0.148	(-0.370 , 0.204)
METLOC3	-0.310	0.123	(-0.551 , -0.067)	METLOC3	0.061	0.137	(-0.199 , 0.331)
METLOC4	0.419	0.194	(0.032 , 0.795)	METLOC4	0.638	0.222	(0.210 , 1.079)
METLOC5	0.425	0.204	(0.027 , 0.820)	METLOC5	0.108	0.227	(-0.333 , 0.553)
BSDTARG	0.008	0.001	(0.005 , 0.010)	BSDTARG	0.003	0.002	(0.000 , 0.006)
R	-0.320	0.159	(-0.615 , 0.006)	R	-0.048	0.143	(-0.317 , 0.243)
$\mathrm{V}_{\mathrm{T_P}}$	0.962	0.140	(0.700 , 1.249)	β_1	0.038	0.047	(-0.057 , 0.130)
$\mathrm{V}_{\mathrm{T_{SW}}}$	-0.282	0.197	(-0.679 , 0.093)	Longitudinal			
$\mathrm{R} * \mathrm{V}_{\mathrm{T_{SW}}}$	0.300	0.226	(-0.137 , 0.733)	θ_1	8.533	0.256	(8.041 , 9.032)
β_2	-0.162	0.039	(-0.237 , -0.087)	θ_2	-0.076	0.019	(-0.113 , -0.037)
Logistic				ECOG	-0.612	0.169	(-0.939 , -0.284)
Intercept	3.076	1.214	(1.151 , 5.695)	AGE	0.119	0.181	(-0.222 , 0.484)
ECOG	0.040	0.538	(-2.041 , 2.204)	PEGFRAD	-0.144	0.158	(-0.447 , 0.167)
AGE	-0.342	0.887	(-2.553 , 2.716)	Female	0.092	0.220	(-0.345 , 0.509)
PEGFRAD	-0.727	0.948	(-3.318 , 1.349)	PCRTDC	-0.088	0.156	(-0.377 , 0.226)
Female	1.058	1.839	(-2.094 , 5.293)	TMSITED01	-0.945	0.174	(-1.295 , -0.621)
PCRTDC	0.935	1.135	(-1.200 , 3.024)	METLOC2	0.369	0.187	(-0.006 , 0.722)
R	1.833	1.811	(-0.676 , 2.554)	METLOC3	0.136	0.162	(-0.175 , 0.446)
				METLOC4	0.433	0.270	(-0.117 , 0.957)
				METLOC5	0.248	0.291	(-0.322 , 0.818)
				BSDTARG	0.002	0.002	(-0.002 , 0.006)
				R	0.259	0.165	(-0.073 , 0.570)
				Ω_{11}	1.917	0.204	(1.544 , 2.323)
				Ω_{21}	0.057	0.034	(-0.014 , 0.120)
				Ω_{22}	0.009	0.006	(0.000 , 0.021)

Table S.4: Posterior Estimates under the Joint Model with HNSW

Parameter	EST	SD	95%HPD	Parameter	EST	SD	95%HPD
Survival				Progression			
ECOG	0.714	0.138	(0.449 , 0.996)	ECOG	0.014	0.153	(-0.273 , 0.323)
AGE	-0.034	0.129	(-0.289 , 0.211)	AGE	-0.206	0.183	(-0.535 , 0.174)
PEGFRAD	0.075	0.120	(-0.158 , 0.313)	PEGFRAD	0.446	0.140	(0.162 , 0.716)
Female	-0.101	0.166	(-0.448 , 0.202)	Female	-0.083	0.204	(-0.478 , 0.319)
PCRTDC	0.208	0.116	(-0.010 , 0.439)	PCRTDC	0.029	0.145	(-0.252 , 0.311)
TMSITED01	0.420	0.129	(0.170 , 0.677)	TMSITED01	-0.156	0.147	(-0.450 , 0.119)
METLOC2	0.125	0.136	(-0.159 , 0.376)	METLOC2	-0.073	0.152	(-0.353 , 0.242)
METLOC3	-0.277	0.123	(-0.513 , -0.031)	METLOC3	0.055	0.138	(-0.205 , 0.332)
METLOC4	0.455	0.199	(0.063 , 0.835)	METLOC4	0.641	0.219	(0.207 , 1.053)
METLOC5	0.428	0.202	(0.030 , 0.815)	METLOC5	0.122	0.227	(-0.324 , 0.561)
BSDTARG	0.008	0.001	(0.005 , 0.011)	BSDTARG	0.003	0.002	(0.000 , 0.006)
R	-0.339	0.162	(-0.662 , -0.027)	R	-0.034	0.143	(-0.307 , 0.250)
V_{T_P}	0.947	0.143	(0.650 , 1.215)	β_1	-0.023	0.030	(-0.084 , 0.034)
$V_{T_{SW}}$	-0.308	0.200	(-0.699 , 0.081)	Longitudinal			
$R * V_{T_{SW}}$	0.343	0.226	(-0.100 , 0.796)	θ_1	8.613	0.348	(7.954 , 9.281)
β_2	-0.143	0.025	(-0.193 , -0.095)	θ_2	-0.100	0.024	(-0.149 , -0.055)
Logistic				ECOG	-1.327	0.246	(-1.820 , -0.854)
Intercept	3.017	1.154	(0.997 , 5.285)	AGE	0.151	0.245	(-0.327 , 0.633)
ECOG	0.047	0.491	(-1.835 , 2.253)	PEGFRAD	-0.204	0.226	(-0.642 , 0.244)
AGE	-0.286	0.800	(-2.553 , 2.413)	Female	0.189	0.324	(-0.421 , 0.842)
PEGFRAD	-0.784	0.934	(-3.188 , 1.202)	PCRTDC	-0.410	0.224	(-0.840 , 0.021)
Female	0.913	1.598	(-2.008 , 5.091)	TMSITED01	-0.290	0.244	(-0.749 , 0.193)
PCRTDC	1.061	1.228	(-1.079 , 3.357)	METLOC2	0.001	0.264	(-0.507 , 0.526)
R	1.651	1.665	(-0.665 , 2.174)	METLOC3	0.406	0.236	(-0.060 , 0.855)
				METLOC4	1.345	0.389	(0.588 , 2.099)
				METLOC5	-0.088	0.410	(-0.894 , 0.673)
				BSDTARG	0.000	0.003	(-0.005 , 0.006)
				R	-0.070	0.221	(-0.510 , 0.354)
				Ω_{11}	4.580	0.400	(3.858 , 5.399)
				Ω_{21}	0.017	0.064	(-0.110 , 0.141)
				Ω_{22}	0.025	0.013	(0.000 , 0.050)

Table S.5: Posterior Estimates under the Cure Rate Model Alone for Time to Progression

Parameter	EST	SD	95%HPD	Parameter	EST	SD	95%HPD
Progression				Logistic			
ECOG	0.013	0.155	(-0.274 , 0.324)	Intercept	3.063	1.215	(1.107 , 5.566)
AGE	-0.214	0.182	(-0.577 , 0.133)	ECOG	0.060	0.527	(-1.999 , 2.177)
PEGFRAD	0.441	0.139	(0.181 , 0.725)	AGE	-0.265	0.917	(-2.356 , 2.879)
Female	-0.078	0.210	(-0.481 , 0.336)	PEGFRAD	-0.795	1.014	(-3.472 , 1.230)
PCRTDC	0.026	0.146	(-0.248 , 0.328)	Female	1.024	1.832	(-2.071 , 5.316)
TMSITED01	-0.151	0.143	(-0.430 , 0.124)	PCRTDC	0.975	1.034	(-0.903 , 3.250)
METLOC2	-0.072	0.152	(-0.357 , 0.229)	R	1.681	1.731	(-0.602 , 2.392)
METLOC3	0.057	0.138	(-0.210 , 0.328)				
METLOC4	0.635	0.222	(0.189 , 1.051)				
METLOC5	0.121	0.223	(-0.299 , 0.558)				
BSDTARG	0.003	0.002	(-0.001 , 0.006)				
R	-0.033	0.143	(-0.302 , 0.252)				

The posterior estimates were obtained using 25 pieces for the piecewise baseline hazard function. The values of LPML were -594.80, -555.01, and -575.10 and the values of DIC were 1175.93, 1098.03, and 1125.91 for the models with 20, 25, and 30 pieces, respectively.

Table S.6: Posterior Estimates under the Survival Model Alone for Overall Survival

Parameter	EST	SD	95%HPD
ECOG	0.712	0.134	(0.452 , 0.977)
AGE	-0.121	0.124	(-0.367 , 0.115)
PEGFRAD	0.200	0.113	(-0.012 , 0.428)
Female	-0.081	0.157	(-0.406 , 0.205)
PCRTDC	0.258	0.110	(0.046 , 0.478)
TMSITED01	0.367	0.125	(0.117 , 0.607)
METLOC2	0.086	0.134	(-0.181 , 0.344)
METLOC3	-0.273	0.117	(-0.496 , -0.037)
METLOC4	0.552	0.187	(0.192 , 0.912)
METLOC5	0.376	0.193	(-0.007 , 0.746)
BSDTARG	0.008	0.001	(0.005 , 0.010)
R	-0.205	0.115	(-0.434 , 0.020)

The posterior estimates were obtained using 7 pieces for the piecewise baseline hazard function. The values of LPML were -1118.75, -1113.14, and -1116.67 and the values of DIC were 2236.43, 2224.80, and 2231.79 for the models with 5, 7, and 10 pieces, respectively.

Table S.7: Posterior Estimates under the Survival Model Alone for Overall Survival with Time Dependent Covariates

Parameter	EST	SD	95%HPD
ECOG	0.699	0.135	(0.438 , 0.959)
AGE	-0.063	0.124	(-0.312 , 0.171)
PEGFRAD	0.091	0.115	(-0.131 , 0.316)
Female	-0.043	0.158	(-0.342 , 0.279)
PCRTDC	0.194	0.112	(-0.025 , 0.416)
TMSITED01	0.380	0.125	(0.138 , 0.623)
METLOC2	0.119	0.133	(-0.145 , 0.369)
METLOC3	-0.312	0.118	(-0.547 , -0.094)
METLOC4	0.452	0.187	(0.073 , 0.808)
METLOC5	0.353	0.196	(-0.026 , 0.733)
BSDTARG	0.007	0.001	(0.005 , 0.010)
R	-0.355	0.160	(-0.680 , -0.055)
$\mathrm{V}_{\mathrm{T_P}}$	0.938	0.139	(0.672 , 1.216)
$\mathrm{V}_{\mathrm{T_{SW}}}$	-0.353	0.203	(-0.752 , 0.033)
$\mathrm{R} * \mathrm{V}_{\mathrm{T_{SW}}}$	0.315	0.231	(-0.148 , 0.751)

S.5 Additional Figures

Survival

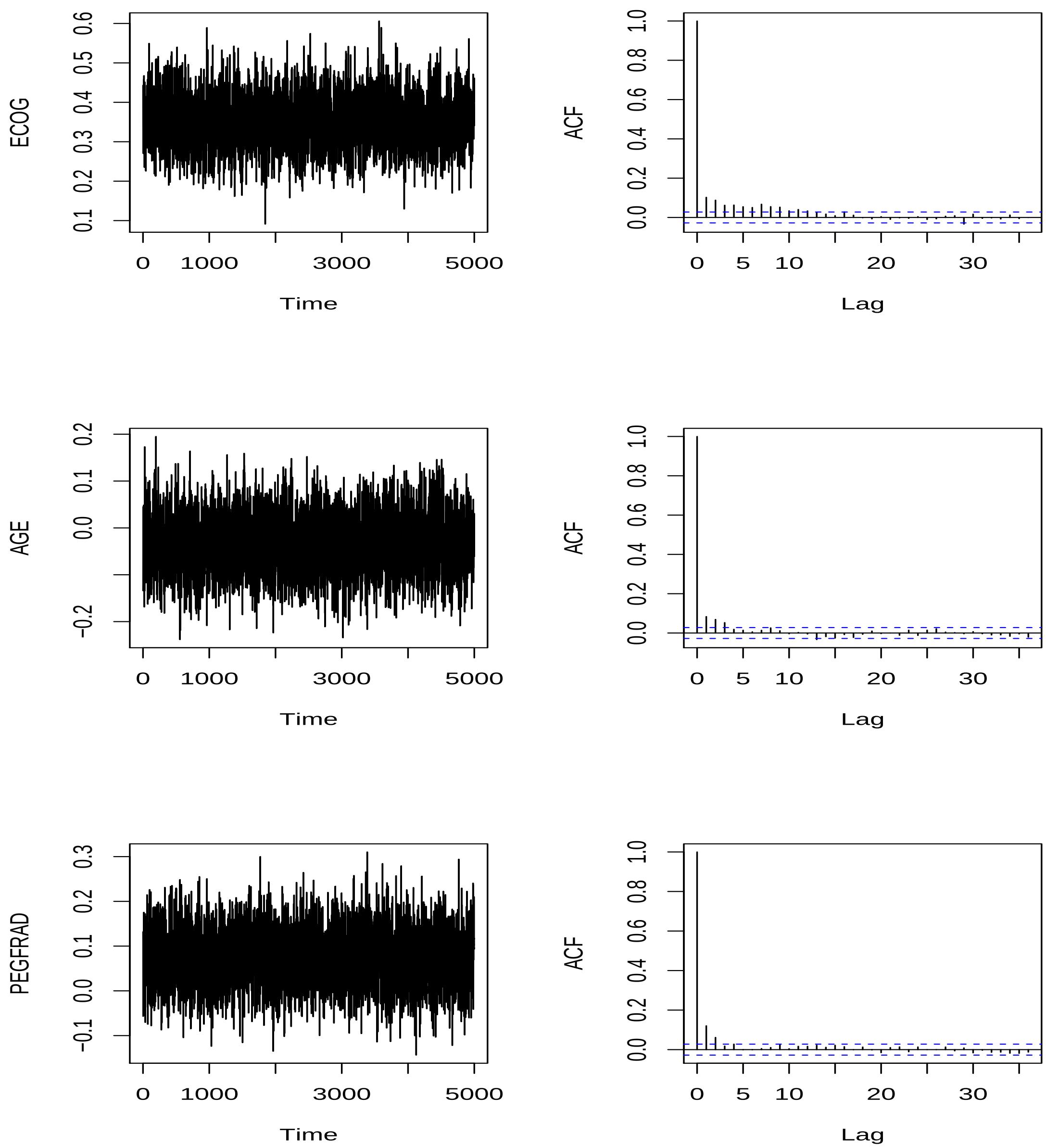

Figure S.1: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to ECOG, AGE, and PEGFRAD under the overall survival model (3.10).

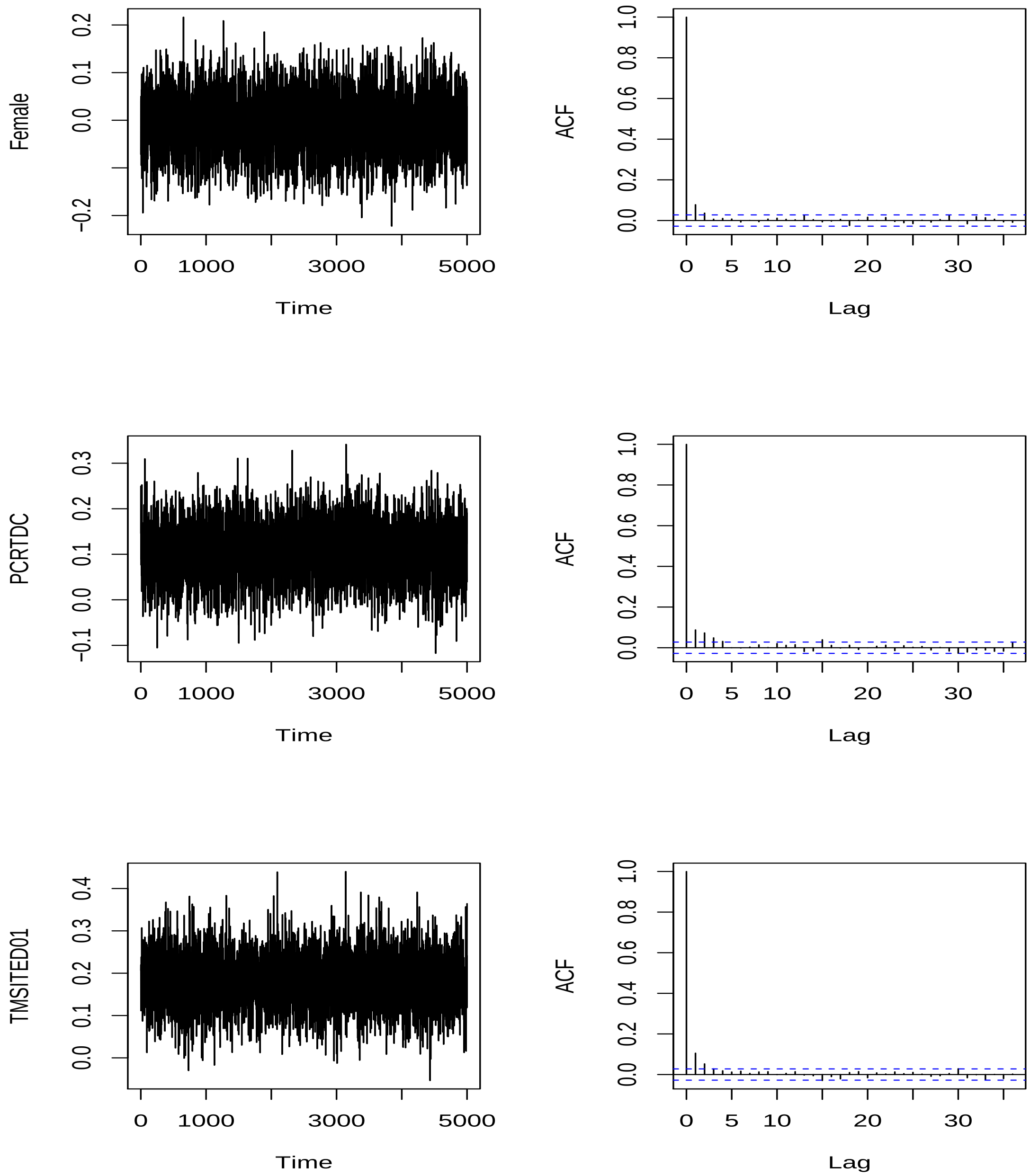

Figure S.2: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to Female, PCRTDC, and TMSITED01 under the overall survival model (3.10).

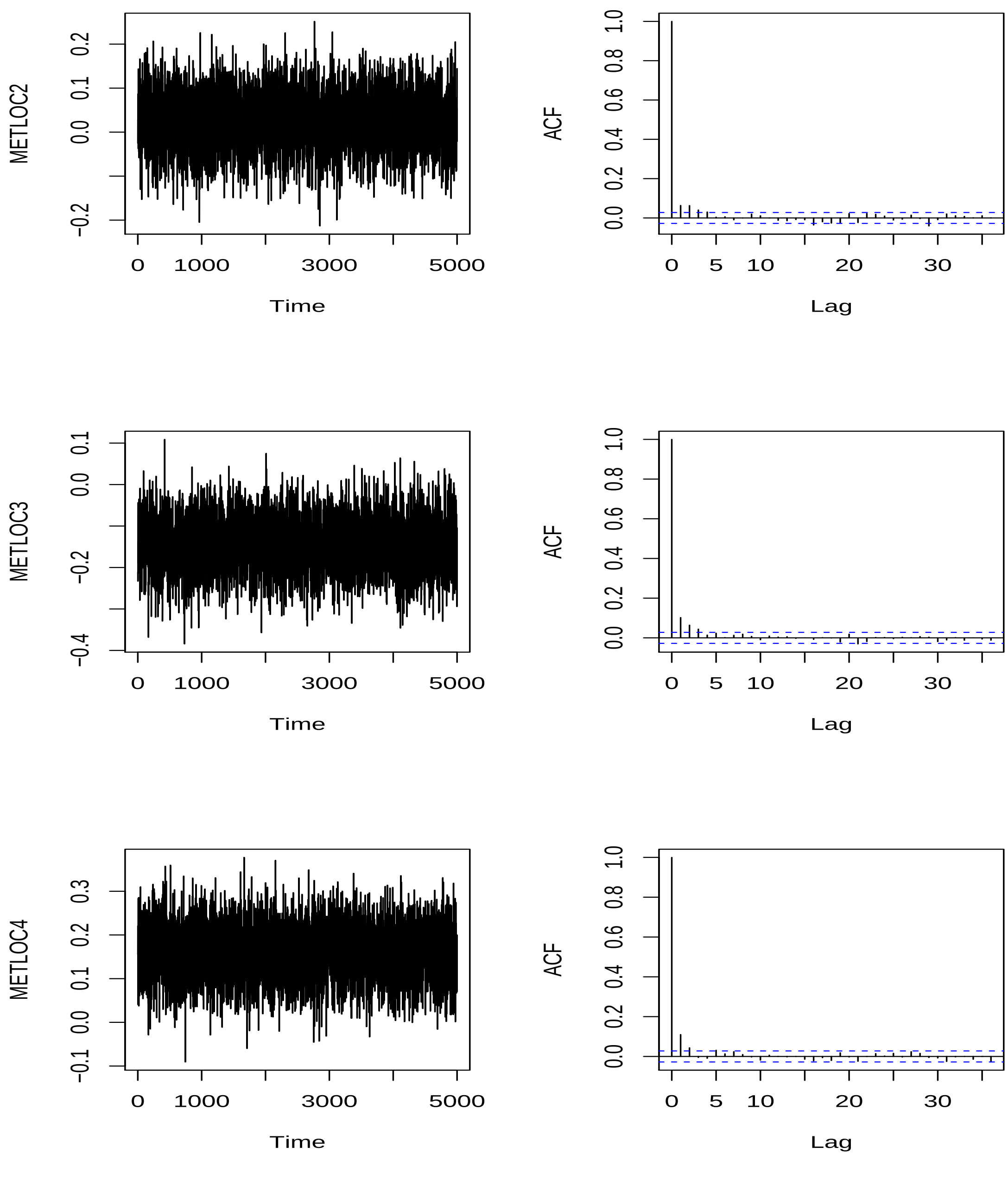

Figure S.3: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to METLOC2, METLOC3, and METLOC4 under the overall survival model (3.10).

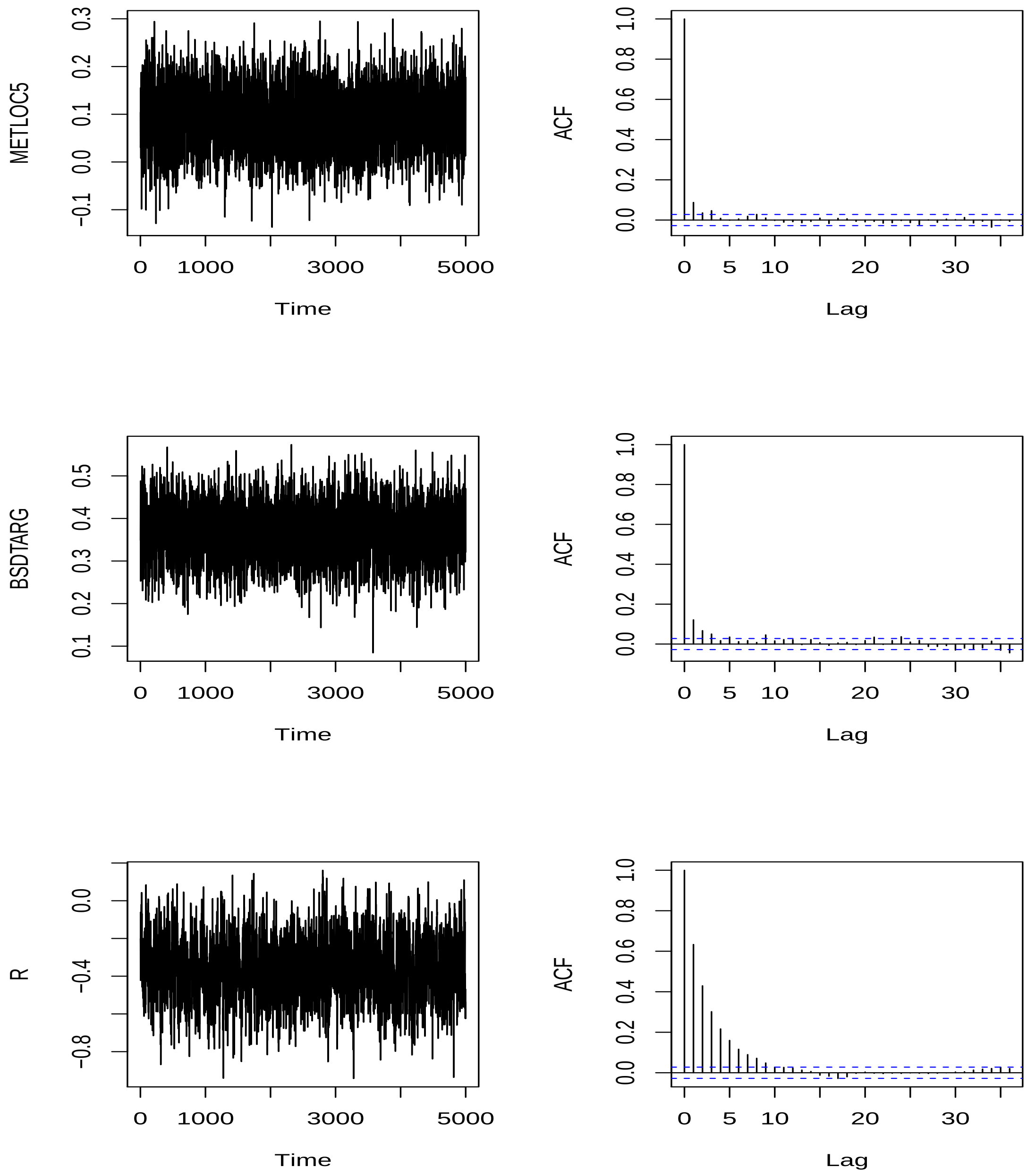

Figure S.4: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to METLOC5, BSDTARG, and R under the overall survival model (3.10).

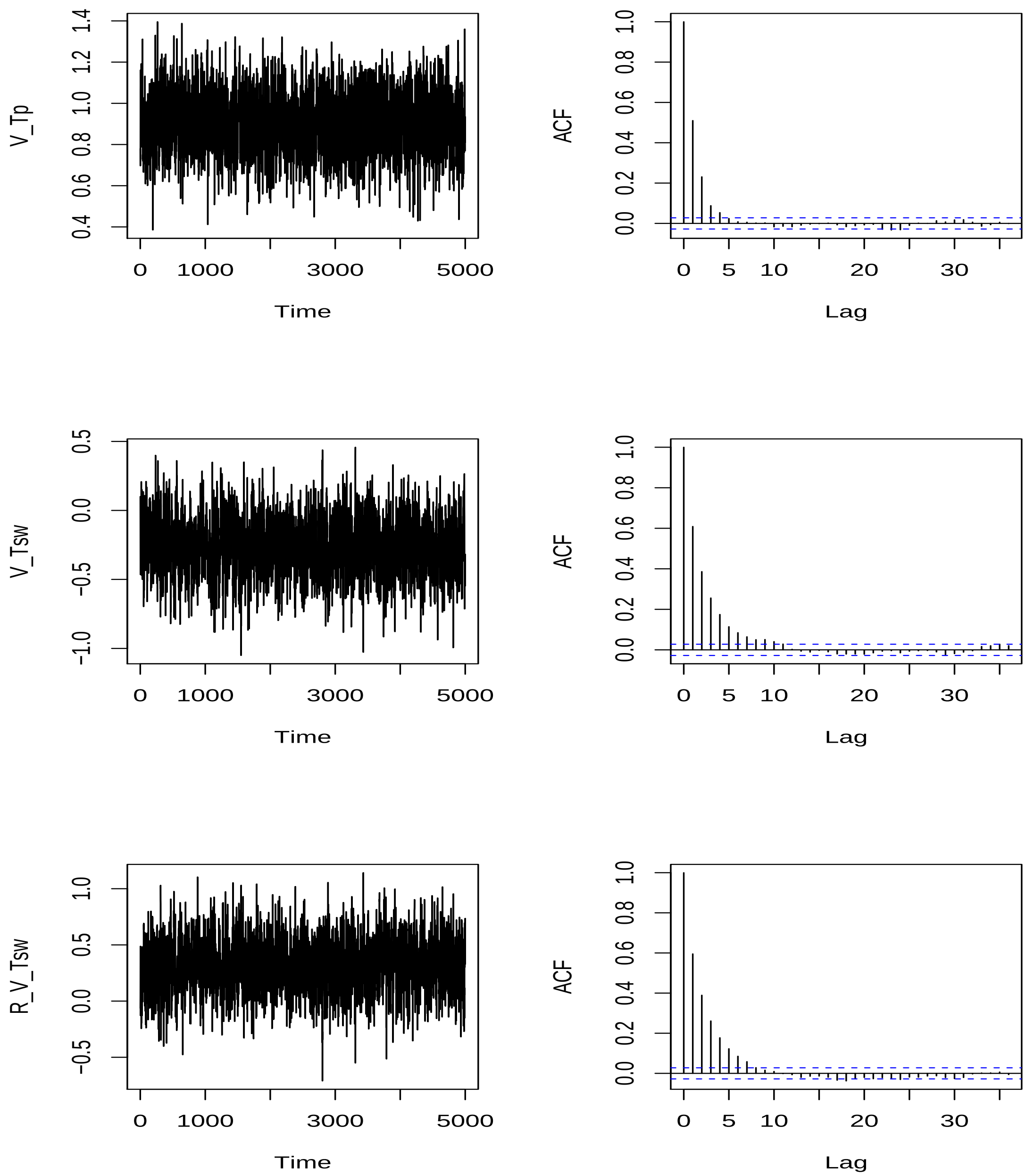

Figure S.5: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to $\mathrm{V_{T_P}}$, $\mathrm{V_{T_{SW}}}$, and $\mathrm{R * V_{T_{SW}}}$ under the overall survival model (3.10).

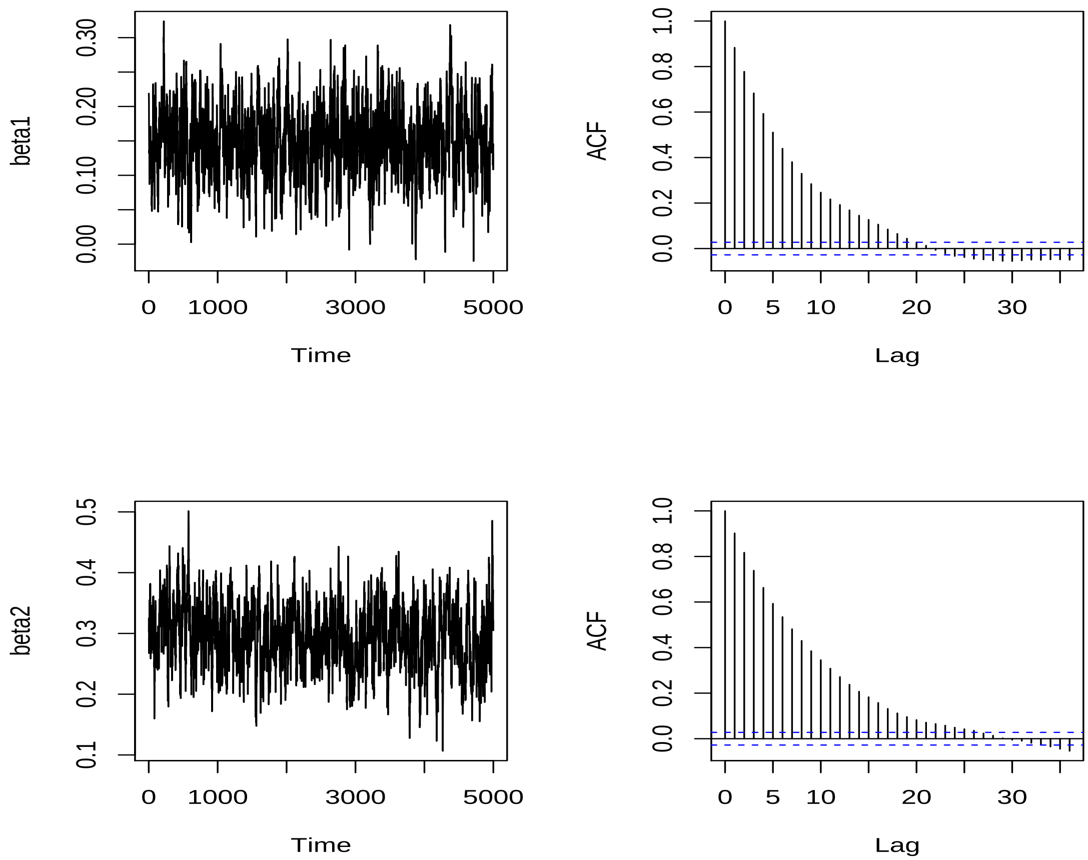

Figure S.6: Trace and autocorrlation plots of MCMC chains for β_1 under the time to progression model (3.8) and β_2 under the overall survival model (3.10).

Progression

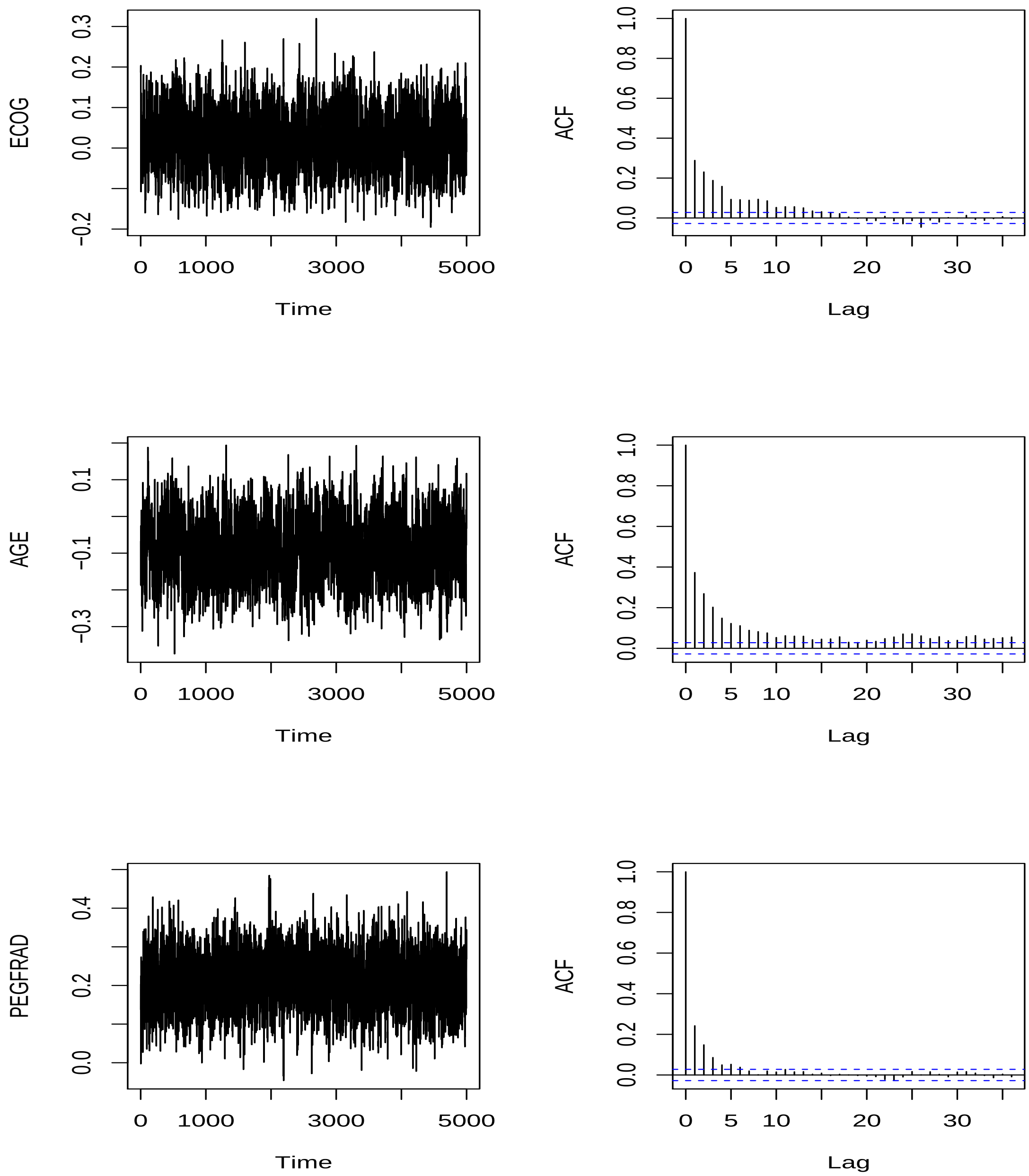

Figure S.7: Trace and Autocorrlation Plots of MCMC chains for the regression coefficients corresponding to ECOG, AGE, and PEGFRAD under the time to progression model (3.8).

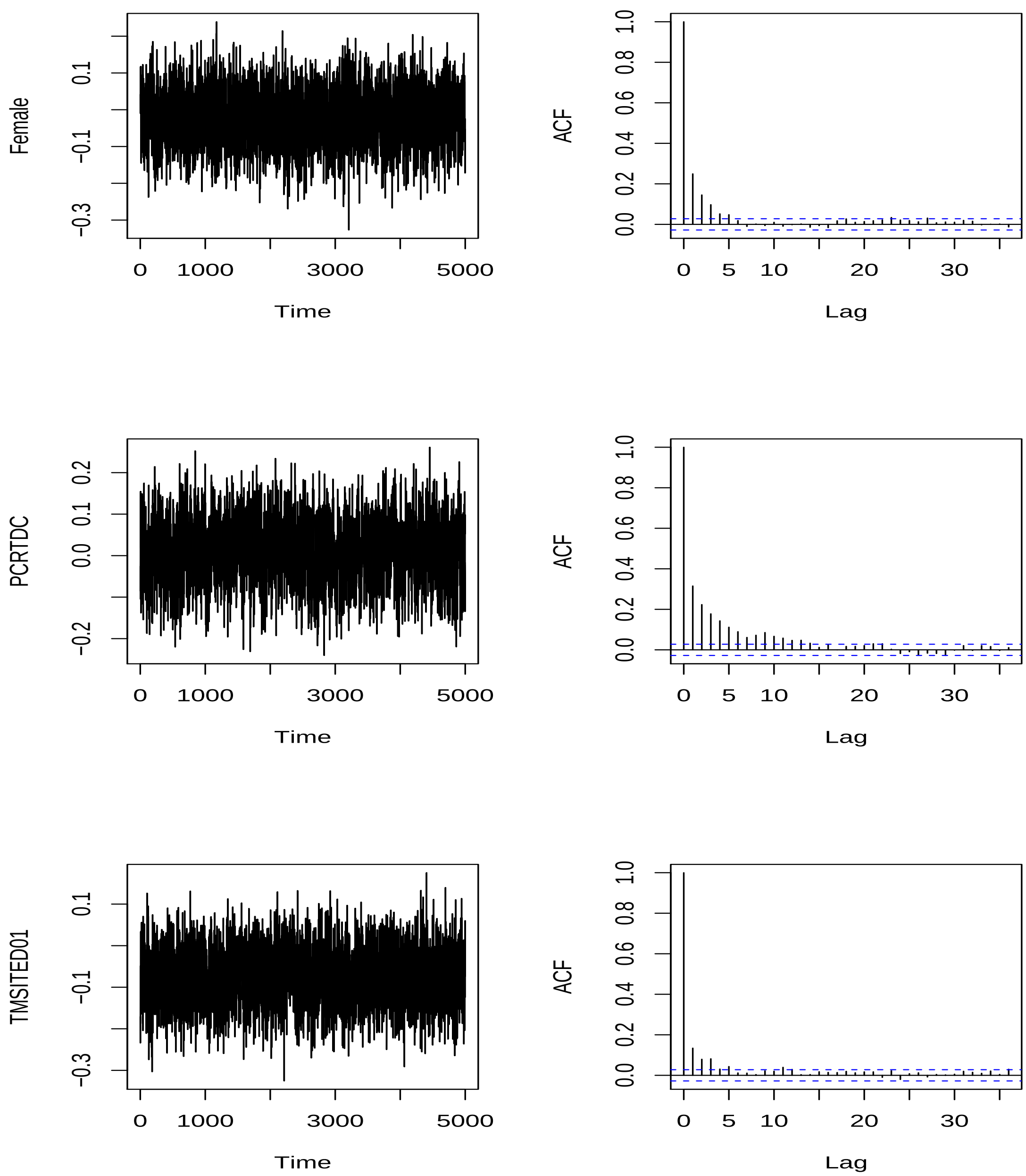

Figure S.8: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to Female, PCRTDC, TMSITED01 under the time to progression model (3.8).

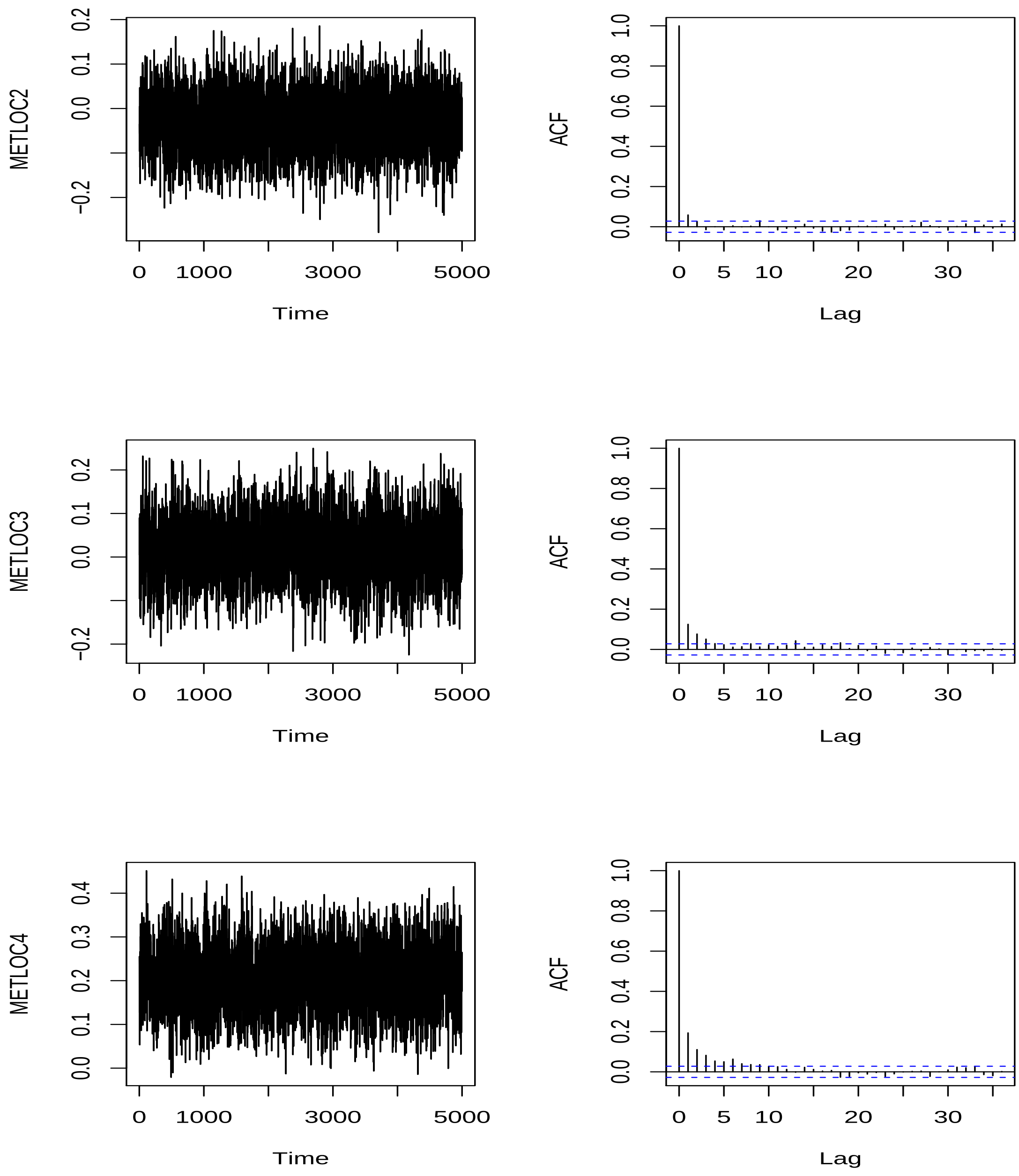

Figure S.9: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to METLOC2, METLOC3, and METLOC4 under the time to progression model (3.8).

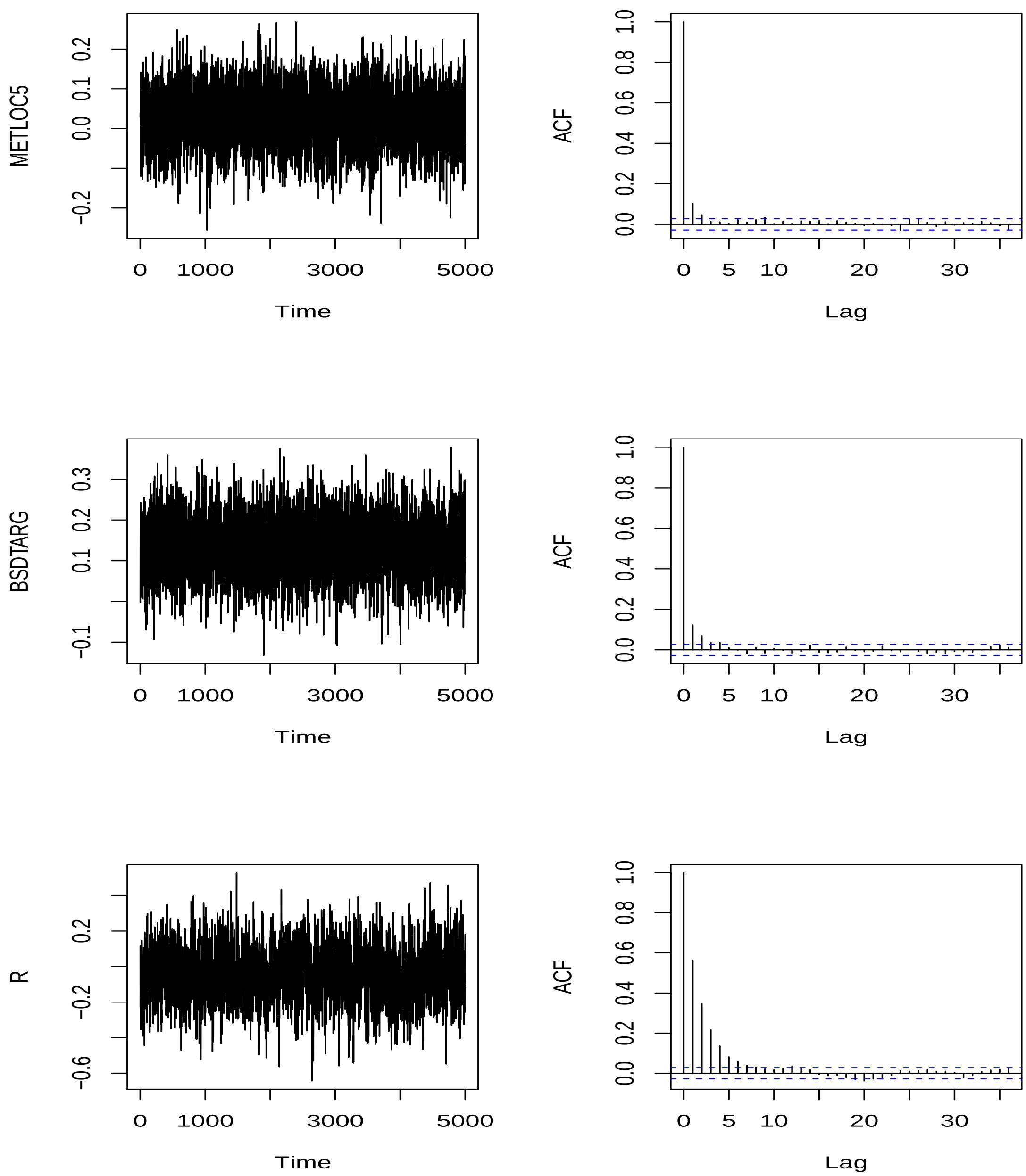

Figure S.10: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to METLOC5, BSDTARG, and R under the time to progression model (3.8).

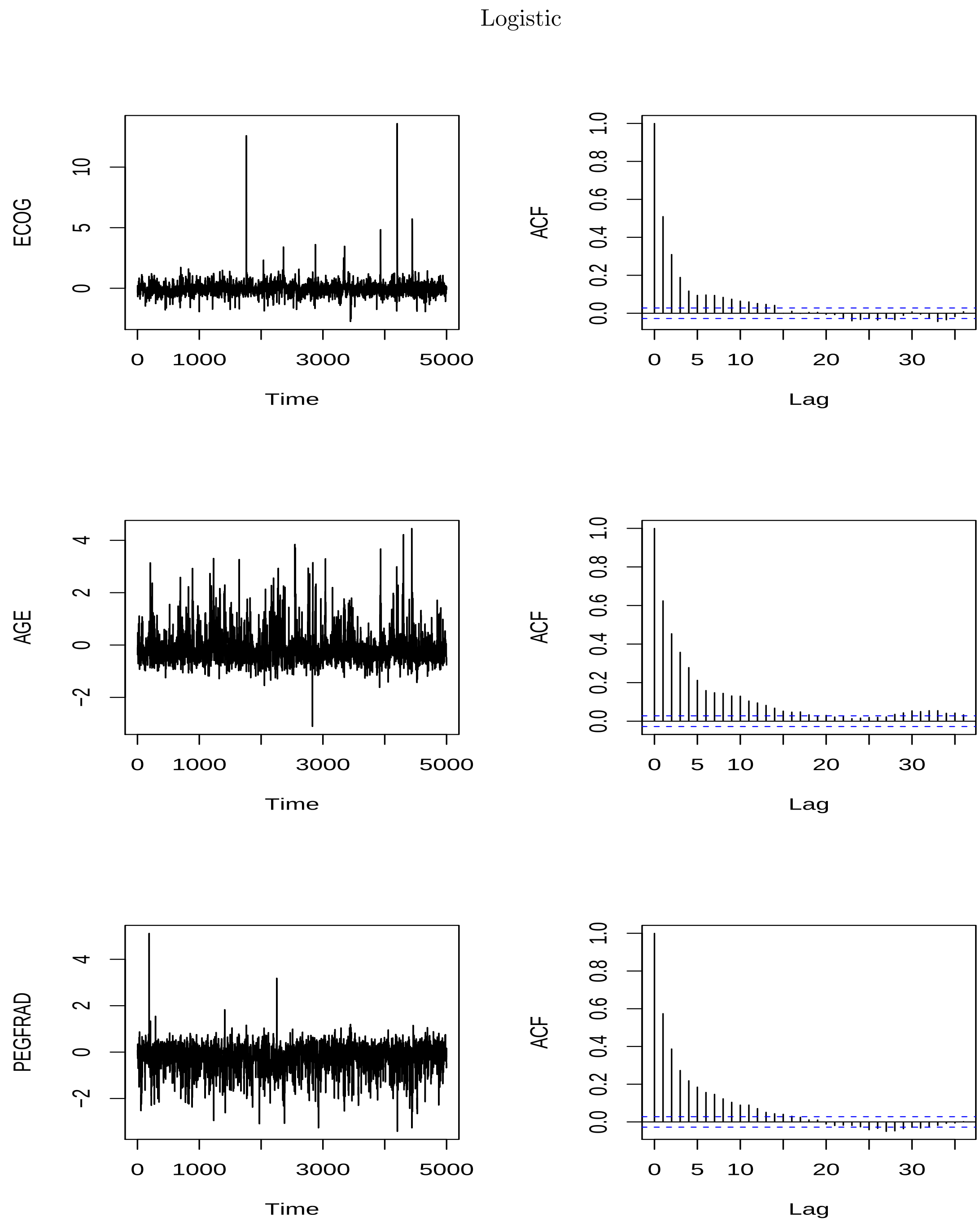

Figure S.11: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to ECOG, AGE, and PEGFRAD under the logistic regression model (3.7).

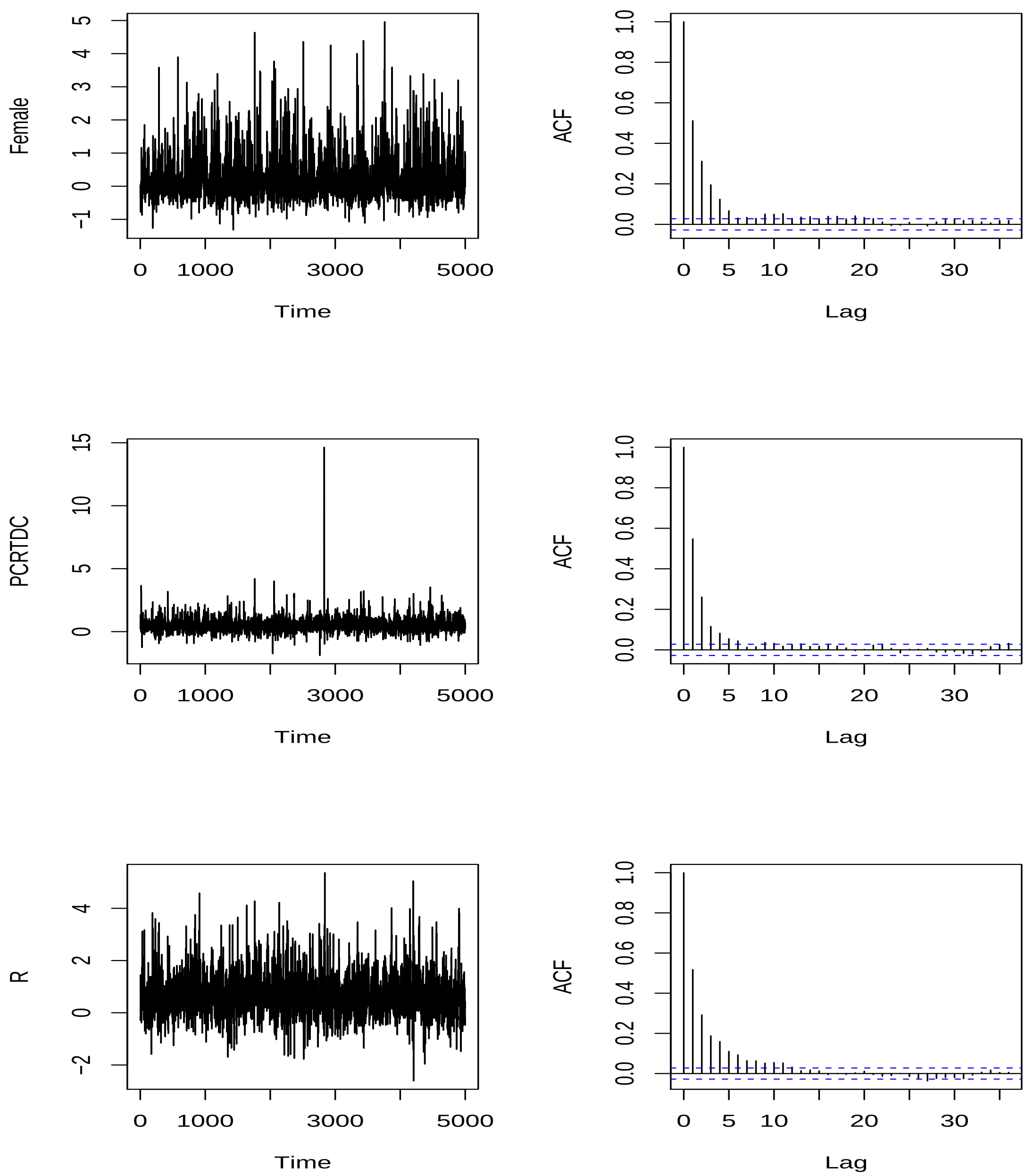

Figure S.12: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to Female, PCRTDC, and R under the logistic regression model (3.7).

Longitudinal

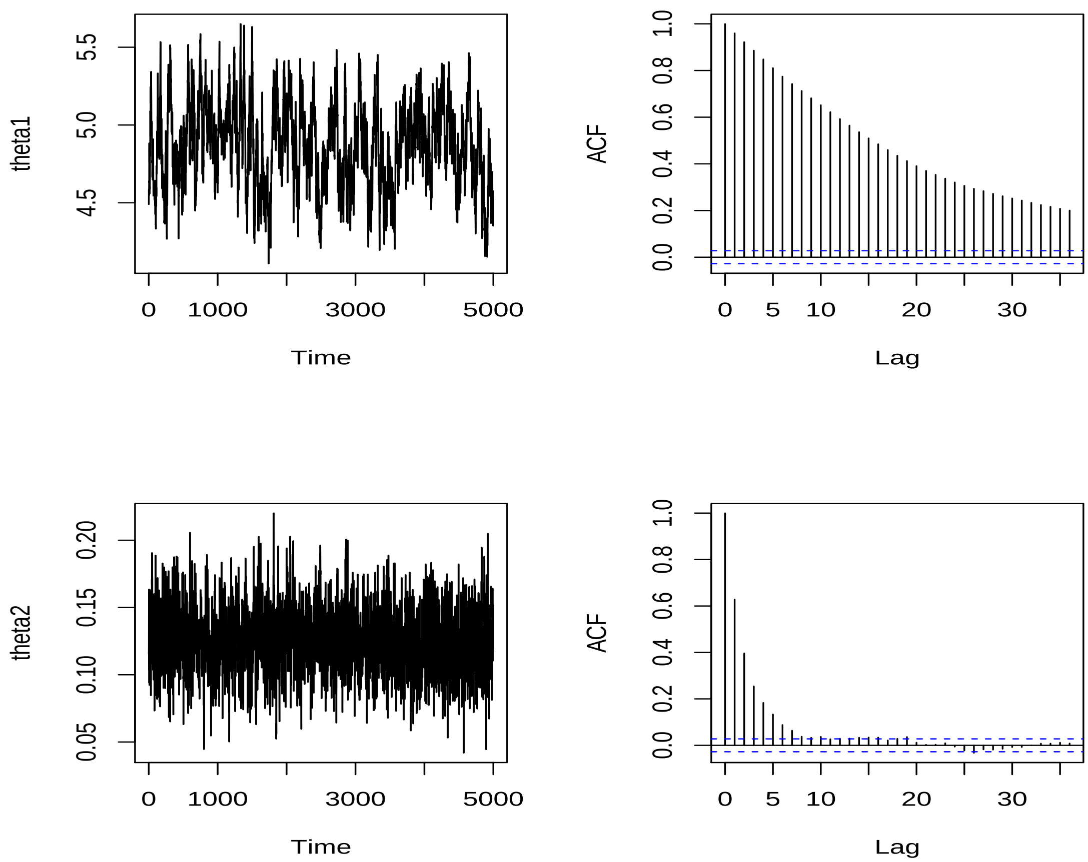

Figure S.13: Trace and autocorrlation plots of MCMC chains for θ_1 and θ_2 under the longitudinal model (3.1).

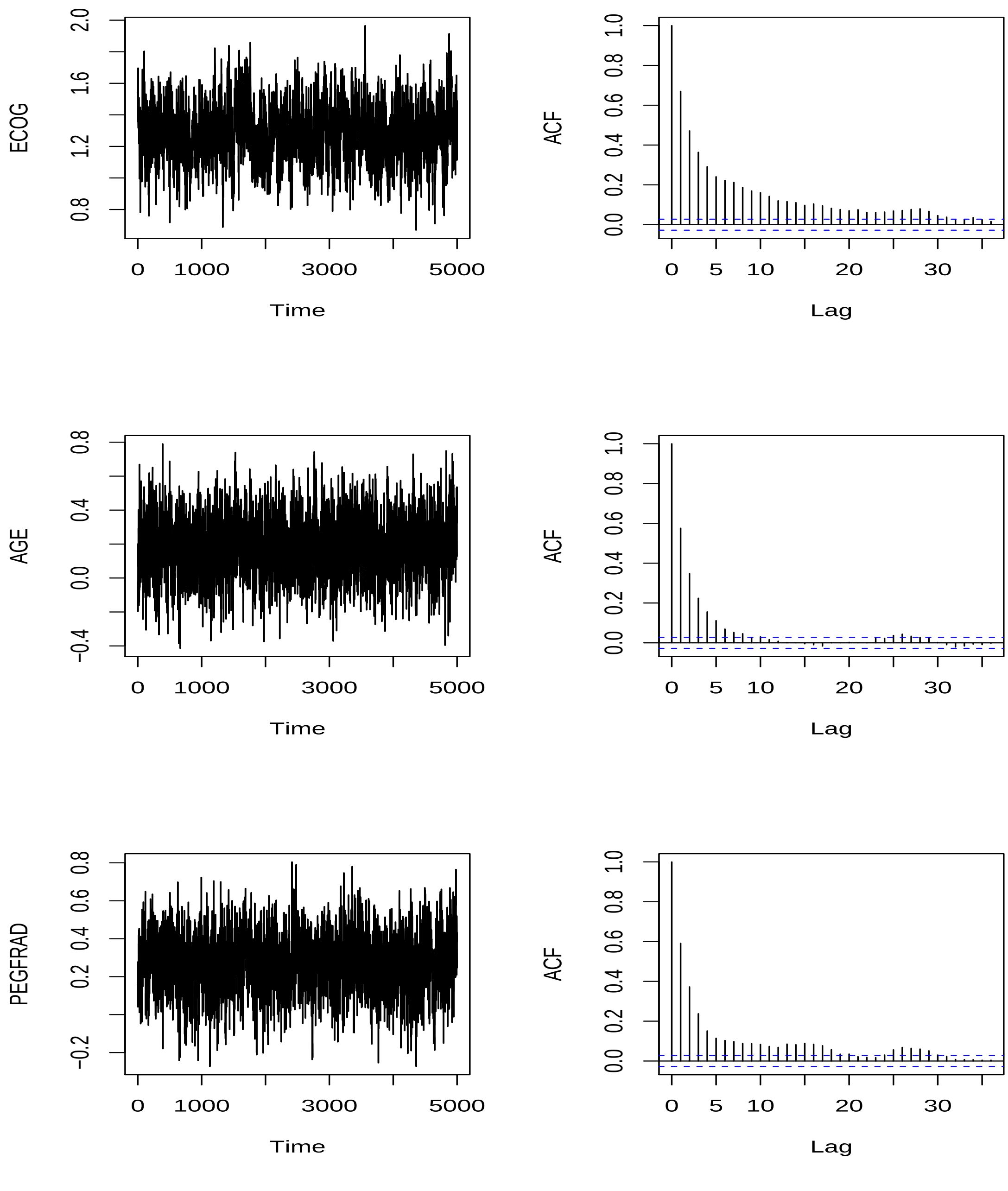

Figure S.14: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to ECOG, AGE, and PEGFRAD under the longitudinal model (3.1)

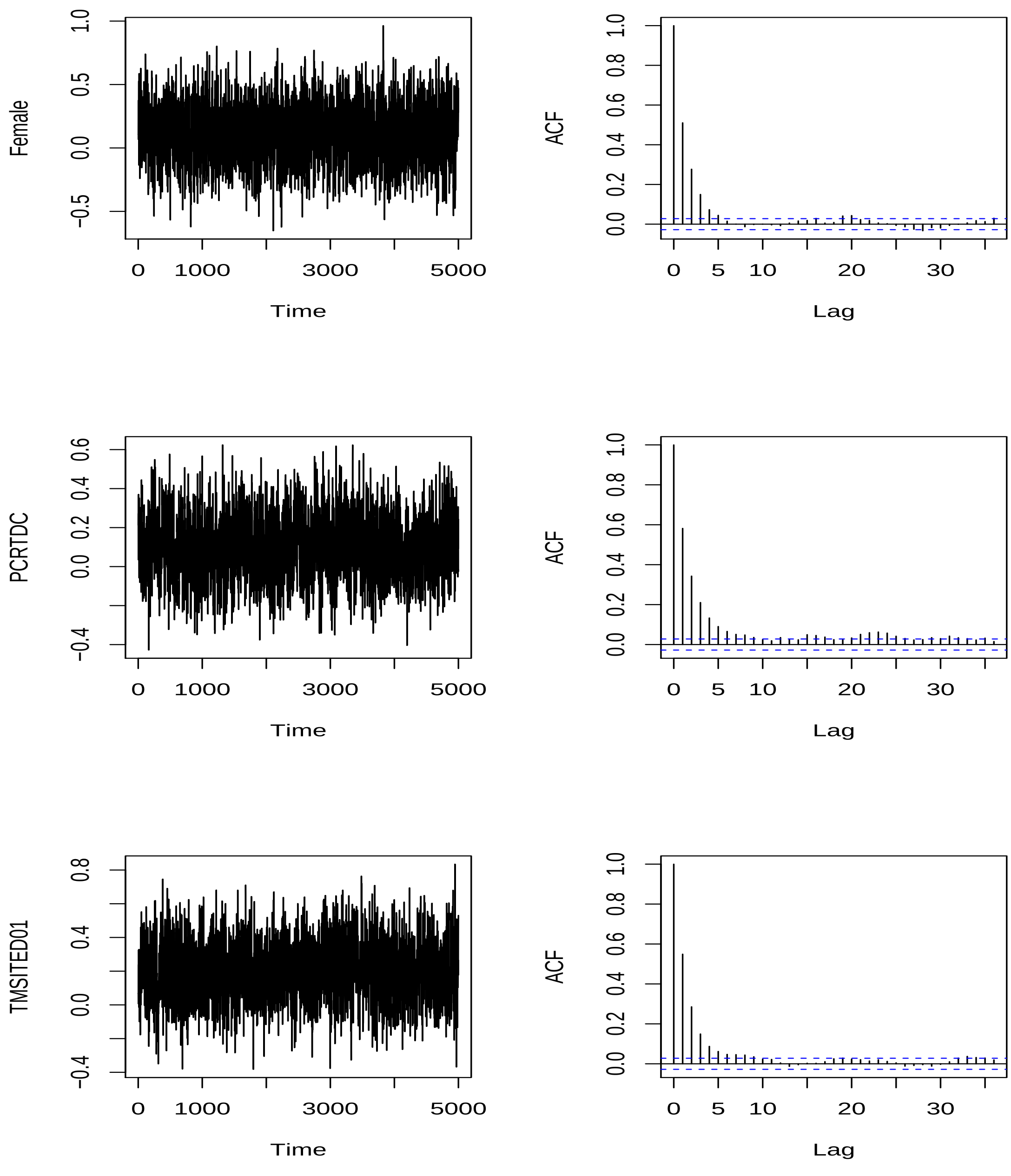

Figure S.15: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to Female, PCRTDC, and TMSITED01 under the longitudinal model (3.1).

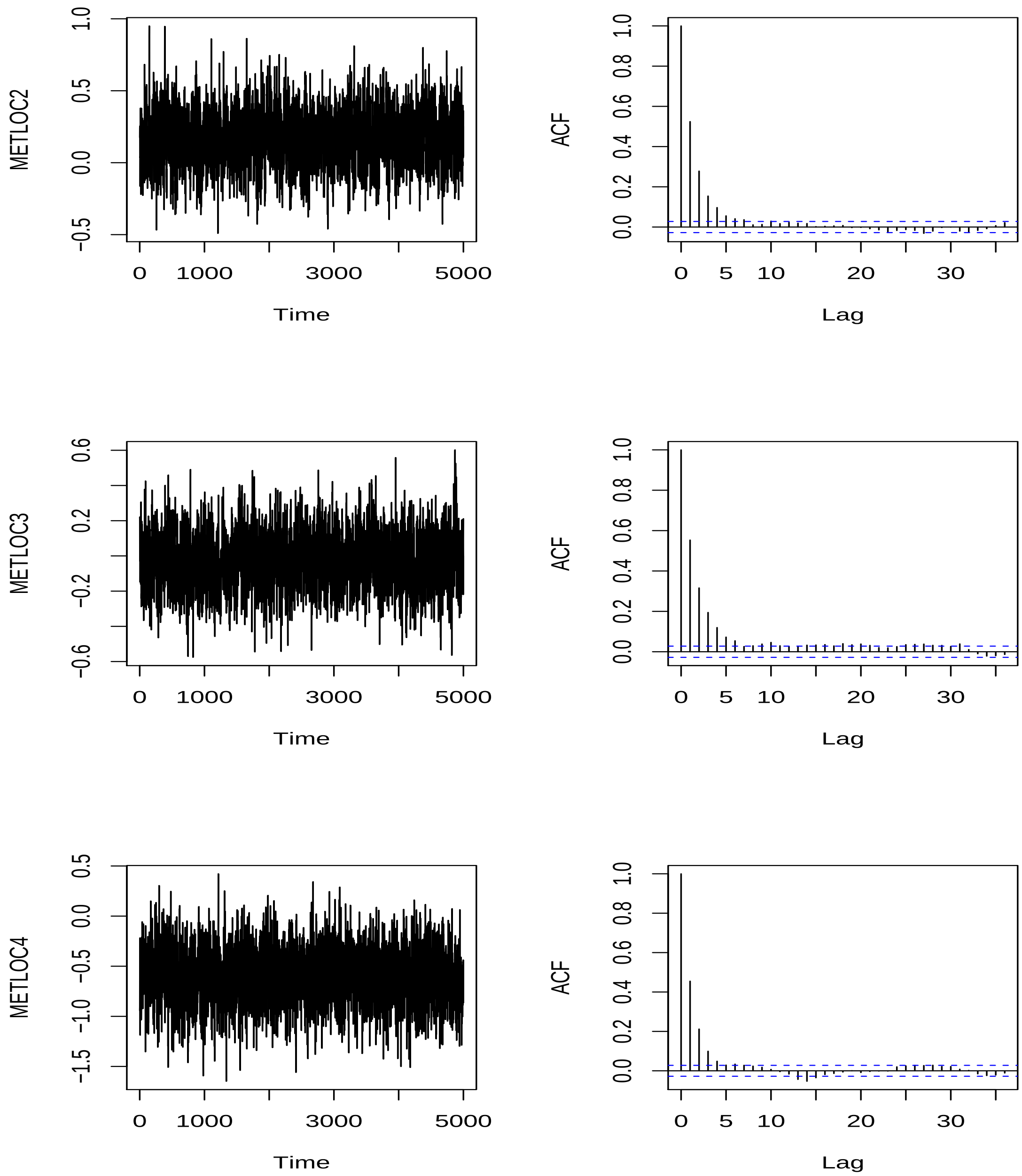

Figure S.16: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to METLOC2, METLOC3, METLOC4 under the longitudinal model (3.1).

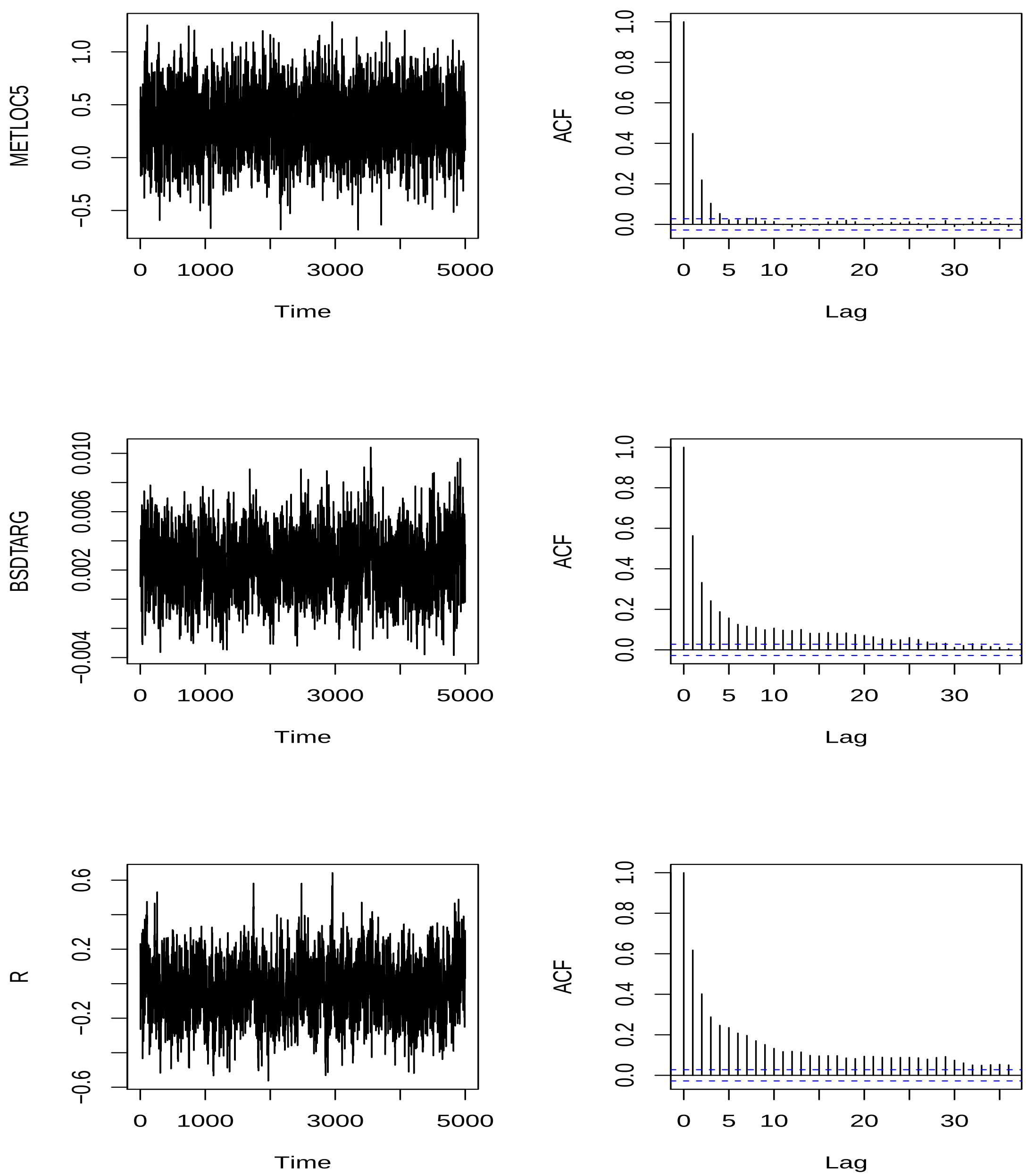

Figure S.17: Trace and autocorrlation plots of MCMC chains for the regression coefficients corresponding to METLOC5, BSDTARG, and R under the longitudinal model (3.1).

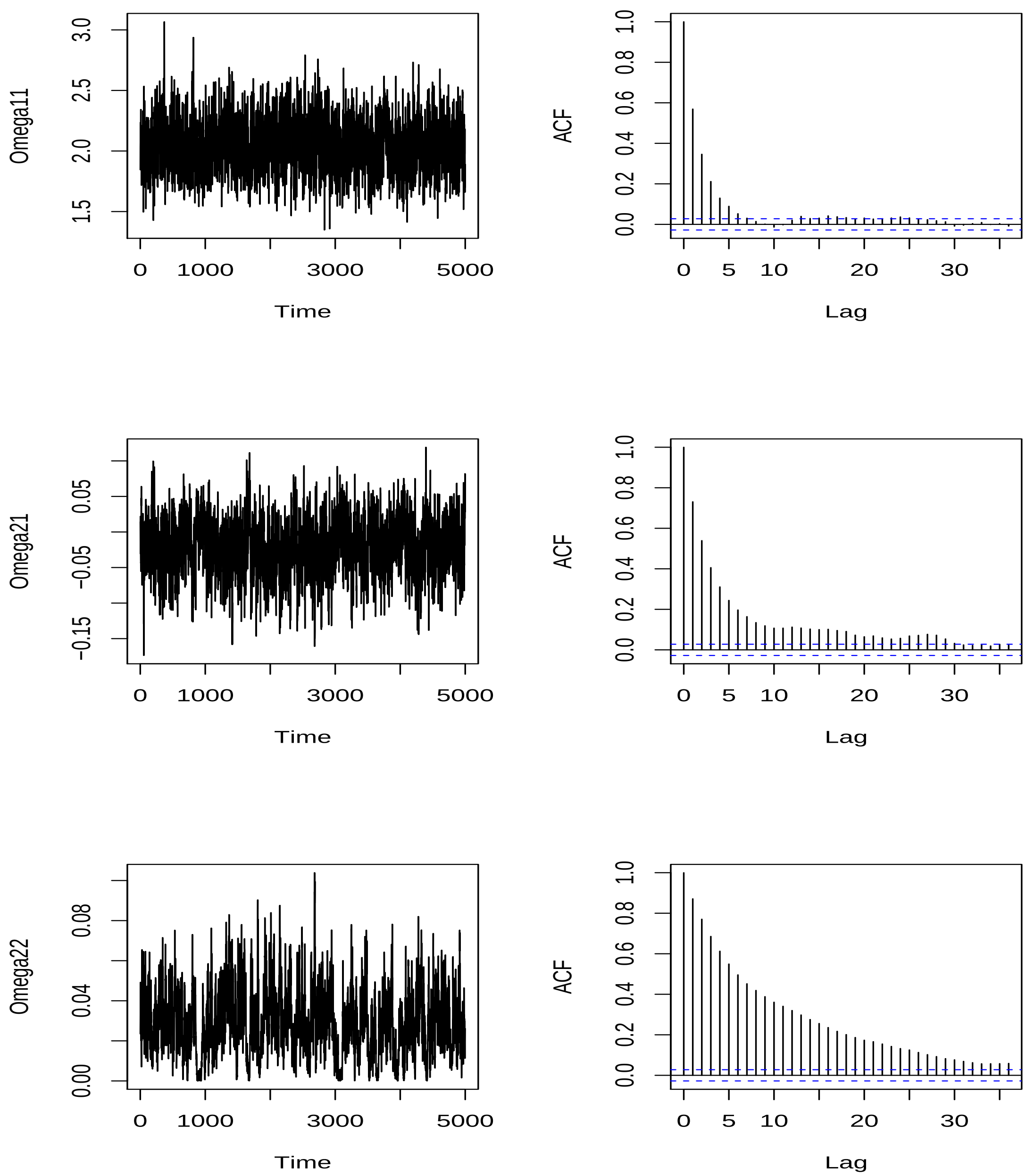

Figure S.18: Trace and autocorrlation plots of MCMC chains for the variance component to Ω_{11}, Ω_{21}, and Ω_{22} under the longitudinal model (3.1).